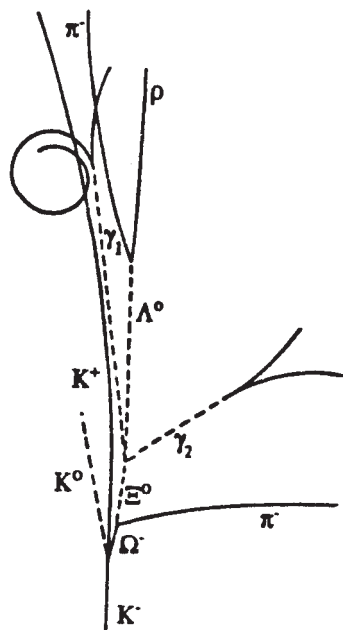


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**SANTILLI'S ISOTOPIC, GENOTOPIC, HYPER-STRUCTURAL AND
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**SANTILLI'S ISOTOPIC, GENOTOPIC, HYPER-STRUCTURAL AND ISODUAL
FOUR DIRECTIONS OF TIME AND NONEQUILIBRIUM THERMODYNAMICS¹****Anil A. Bhalekar²**

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Abstract

The decades of research by Professor R. M. Santilli saw the advent of iso-, isodual-, geno-, genoisodual- and hyper- mathematics and corresponding lifting of mechanics and quantum mechanics (see for example the refs.: [1, 2]). This then removed a multitude of inadequacies existing in various branches of science e.g. general and special relativity, quantum mechanics, quantum chemistry, astrophysics, cosmology, particle physics, nuclear physics, and so on. Thus we can safely say that the new mathematics of Santilli produced new sciences for a new era. Santilli has discovered (see for example the ref.: [3]) for the first time that there are four directions of time's arrow and not two asserted earlier by Eddington [4]. Herein we have briefly reviewed Santilli's *isotimes*, *genotimes* and *genohypertimes*. To this list we have added *genothermodynamic times* identified for the first time by the present author [5] by reanalyzing the genononequilibrium thermodynamics of coupled processes [6].

Keywords: Hadronic mechanics, Isomathematics, Isodual mathematics, Lie-admissible mathematics, Genomathematics, Hyper mathematics, Nonequilibrium thermodynamics, Geno-nonequilibrium thermodynamics.

¹This work was presented at 11th International Conference of Numerical Analysis and Applied Mathematics 2013 (ICNAAM 2013), Rhodes, Greece, 21-27 September 2013 and also at the 2013 International Workshop on Hadronic Chemistry, Mathematics and Physics held at the Department of Chemistry, R. T. M. Nagpur University, Nagpur, India during October 21-26, 2013.

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1 Introduction

The celebrated Eddington's arrow of time [4] has only two directions, forward and backward. Human senses too recognize only past and future directions and hence Eddington's arrow of time is considered as an adequate and proper description commensurate with our physical observations. The dynamical description of a self-evolving system consists of a host of state functions besides the time parameter. However, out of these host of state functions Eddington identified a uniqueness associated with entropy function with respect to time's arrow. Notice should be made of the fact that the entropy function that Eddington is referring is the one which is primarily a thermodynamic quantity but there is a quantity also termed as entropy but basically constructed using the probabilistic approach and has more than one versions, such as Boltzmann entropy, Shannon entropy, Gibbs entropy and so on. As asserted by Dunning-Davies³ [7] it is not clear even today that when and where to use which entropy function out of them.

Based on the second law of thermodynamics inequality (and in recognition of its statistical mechanical character), namely:

$$\Delta S \geq 0 \quad (\text{Adiabatically closed or isolated systems})$$

Eddington's primary assertion is as follows.

- *Nothing in the statistics of an assemblage can distinguish a direction of time when entropy fails to distinguish one.*

Then he further goes on and asserts that,

- *whereas other statistical characters besides entropy might perhaps be used to discriminate time's arrow, they can only succeed when it succeeds and they fail when it fails,*
- *so far as physics is concerned time's arrow is a property of entropy alone.*

³J. Dunning-Davies and D. Sands, This point has been discussed at the 2013 International Workshop on Hadronic Chemistry, Mathematics and Physics, October 21-26, 2013, Department of Chemistry, R. T. M. Nagpur University, Nagpur, India.

The above are *the sweeping assertions* that *identifies a special status that entropy function enjoys* and illustrates what actually is the *basic content of the term thermodynamic irreversibility*. Notice that *the mathematics used herein is the conventional Lie algebra that uses conventional numbers*. However, *with the advent of Santilli's iso-, geno- and hyper-mathematics and their isoduals the scientific scenario particularly the physics had changed dramatically*. Hence in the present study *we have focused on the directions of time's arrow as identified by Santilli himself* that got identified with the advent of his new mathematics. And its *thermodynamic implications stressed by the present author [5] are also discussed*. A brief description of the subject matter of this paper has recently appeared [8].

2 Four directions of time

In the scientific history, for the first time, Santilli has identified four directions of time by using iso-, geno-, hyper- and their isodual- mathematics. These remained unfortunately submerged just because the 20th century Lie algebra was incapable of unearthing the two additional directions of time.

2.1 Isotopic liftings

Santilli's Isomathematics (see for example: [9] and [10]) involves the non-unitary transforms that is the traditional right and left multiplying unit, $I = 1$ is replaced by an *isounit*, $0 < \hat{I} = U \times U^\dagger = 1/\hat{T} \neq 1$, where $\hat{T}$ is termed as an *isotopic element*. Hence a generic quantity, A , is isotopically lifted as,

$$\begin{aligned} A(t, r, p, \psi, \psi^\dagger, \dots) &\longrightarrow \hat{A}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \hat{\psi}^\dagger, \dots) \\ &= U \times A(t, r, p, \psi, \psi^\dagger, \dots) \times U^\dagger \end{aligned} \quad (1)$$

and the traditional multiplication gets isotopically lifted as,

$$\begin{aligned} A \times B &\longrightarrow U \times (A \times B) \times U^\dagger \\ &= (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times B \times U^\dagger) \\ &= \hat{A} \times \hat{T} \times \hat{B} = \hat{A} \hat{\times} \hat{B} \end{aligned} \quad (2)$$

Notice that the *non-unitary isotopic element*, $\hat{T}$, distinguishes the traditional multiplication $\times$ from the *iso-multiplication* $\hat{\times} = \times \hat{T} \times$.

2.2 Isotopic law for numerical values of functions and time

Thus according to Santilli's isomathematics a conventional function $f(r)$ is *isotopically lifted* [1] as,

$$f(r) \longrightarrow \hat{f}(\hat{r}) = \hat{I} \times f(r) = \hat{I} \times f(\hat{r} \times \hat{T}) \quad (3)$$

and its projection in the original space reads as,

$$f(r \times \hat{T}) \quad (4)$$

Its interpretation is that [1] while isotopically lifting $f(r)$ it preserves its numerical value over the isofield $\hat{F}$ as $\hat{f}(\hat{r})$ on the isospace, $\hat{S}$, because the variable $\hat{r}$ is multiplied by $\hat{T}$ while the unit to which such a variable is referred to is multiplied by the inverse amount $\hat{I} = 1/\hat{T}$. All the numerical differences emerge in the *projection* of $\hat{f}(\hat{r})$ in the original space (c.f. eq.(4)).

Thus when a function say $f(r)$ is isotopically lifted to $\hat{f}(\hat{r})$ the functions $f(r)$ and $\hat{f}(\hat{r})$ both have identically same numerical values. This is termed as the *Isotopic Law for numerical values* [1, 9, 11].

On the same lines time t gets isotopically lifted to the *isotime* $\hat{t}$ but now it has its own *isounit*, $\hat{I}_{\hat{t}} = 1/\hat{T}_{\hat{t}} > 0$. However, in view of the above stated *Isotopic Law for numerical values* we have numerical equivalence of the conventional time, t_{ext} , and the isotopic time, $\hat{t}_{intr}$, which is intrinsic to the corresponding isospace. Hence, we have,

$$t_{ext} = t_{ext} \times 1 = \hat{t}_{intr} = t_{intr} \times \hat{I}_{\hat{t}} \quad (5)$$

From eq.(5) it is clear that the pace of t_{intr} is not the same as that of t of our universe, t_{ext} . That is when $\hat{I}_{\hat{t}} > 1$ there we have a slower pace of t_{intr} than t_{ext} ($t_{intr} < t_{ext}$) and when $\hat{I}_{\hat{t}} < 1$ there we have faster pace of t_{intr} than t_{ext} ($t_{intr} > t_{ext}$). The practical implication of above stated law is shown in Figure 1.

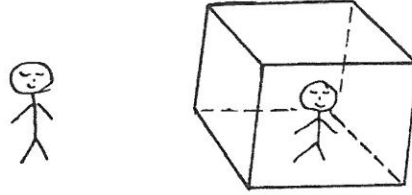


Figure 1: Santilli's illustration of Isospace and Isotime. The observer outside of the box is in real space and real time whereas the box represents isospace with its isotime and hence the observer inside the box sees isospace and his time is isotime.

An observer outside the isobox of Figure 1 senses the pace of the processes (both inside and outside of the isobox) in accordance with the pace of t_{ext} . Whereas an observer inside the isobox senses the pace of the processes (both inside and outside of the isobox) in accordance with the pace of t_{intr} . But in reality the processes within the isobox evolve with the pace of t_{intr} independent of the pace determined by t_{ext} . And the same independence is true for the processes occurring outside the isobox, that is their pace is determined exclusively by t_{ext} which is independent of what is going on within the isobox and its pace is determined by t_{intr} . *This is a vitally important discovery by Santilli having far reaching implications say for example for understanding the time evolution of biological complex systems such as seashells.*

2.3 Directions of time based on iso- and isodual- mathematics

However, with the advent of Santilli's isodual mathematics [12] it becomes evident that each physical quantity has its *anti-isomorphic map* called *isodual*. Thus, now we are led to the isodual Lie algebra and hence in isodual setting evidently there would be corresponding future and past directions. Let us elaborate it a little further. Let us consider an arbitrary quantity A

dependent on say $t, r, p, \psi, \psi^\dagger, \dots$. Its *isodual map* is described as,

$$\begin{aligned} A(t, r, p, \psi, \psi^\dagger, \dots) &\rightarrow A^d(t^d, r^d, p^d, \psi^d, \psi^{\dagger d}, \dots) \\ &= -A^\dagger(-t, -r^\dagger, -p^\dagger, -\psi^\dagger, -\psi, \dots) \end{aligned} \quad (6)$$

The basic quantity of isodual mathematics is then the *isodual isounit*, namely:

$$\hat{I}^d = -\hat{I}^\dagger(-t, -r^\dagger, -p^\dagger, -\psi^\dagger, -\partial\psi^\dagger, \dots) = \frac{1}{\hat{T}^d} < 0 \quad (7)$$

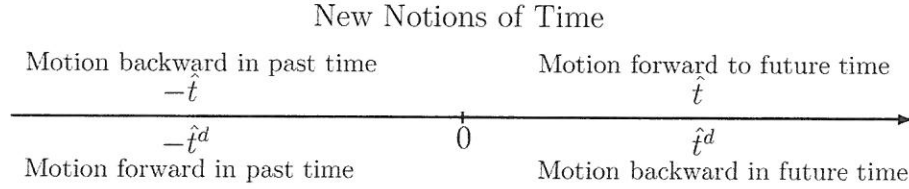


Figure 2: A schematic view of the four possible “time’s directions”.

The *isodual isonumbers* are given by,

$$\hat{a}^d = -\hat{a}^\dagger = -\hat{I}^\dagger \times a \quad (8)$$

Now if a of eq. (8) is time t then the *isotime*, $\hat{t}$ is defined as [9, 11],

$$\hat{t} = t \times \hat{I}_t, \quad 0 < \hat{I}_t \neq 1 \quad (9)$$

and then the isodual time $\hat{t}^d$ gets defined as,

$$\hat{t}^d = t \times \hat{I}_{t^d}^d < 0, \quad \hat{I}_{t^d}^d < 0 \quad (10)$$

Notice that ($\hat{t} > 0$) corresponds to future and describes forward motion whereas ($\hat{t} < 0$) corresponds to past and describes backward motion. Similarly in *isodual space* $\hat{t}^d$ has future ($\hat{t}^d > 0$) and past ($\hat{t}^d < 0$) values. But when we view the latter times through our senses the *future isodual time* describes backward motion in future time and the *past isodual time* describes forward motion in past times. This we depict in Figure 2,

2.4 Anti-isomorphic map of isotopic law of numerical values of functions and time

However, the *anti-isomorphic map* of the *isotopic law of numerical values for time* then reads as [8, 11],

$$t_{ext} = t_{ext} \times 1 = -\hat{t}_{intr}^d = -t_{intr} \times \hat{I}_{t_{intr}}^d \quad (11)$$

Thus we see that if $\hat{I}_{t_{intr}}^d > -1$ we have $t_{ext} < |t_{intr}|$ and if $\hat{I}_{t_{intr}}^d < -1$ we have $t_{ext} > |t_{intr}|$. Therefore, the pace of time in isodual space would be either higher or less than that in our time space. We recall that the *isodual mathematics* has been constructed primarily for describing the dynamics of *antimatter*.

Although, Santilli through his iso- and isodual- mathematics identified, for the first time, four directions of time as described above but these times follow the time reversal symmetry. That is the forward motion becomes backward motion on reversing the sign of time. Therefore, these four directions of time are not suitable for describing irreversible motions as there is no possibility of getting them reversed in the time frame [1, 8]. This drawback from the point of view of irreversibility has been removed by Santilli through his geno- and genoisodual- mathematics.

2.5 Directions of time based on geno- and genoisodual-mathematics

To correctly describe irreversibility of motion Santilli formulated genomathematics and its isodual branch. In genomathematics forward and backward motions are described using different multiplying *genounits* for forward and backward motions [13–15].

The first $\hat{I}^>$, the *forward genounit*, for the *ordered multiplication to the right* $A > B$, called *forward genoproduct*, and the second $\hat{I}^<$, the *backward genounit*, for the *ordered multiplication to the left* $A < B$, called *backward genoproduct*, according to the general rules,

$$\begin{aligned} \hat{I}^> = 1/\hat{S}, A > B &= A \times \hat{S} \times B, \hat{I}^> > A = A > \hat{I}^> = A, \\ \hat{I}^< = 1/\hat{R}, A < B &= A \times \hat{R} \times B, \hat{I}^< < A = A < \hat{I}^< = A \end{aligned} \quad (12)$$

where $\hat{S}$ and $\hat{R}$ are respective *genotopic elements*, that is for the forward and backward multiplications respectively.

Moreover, it is assumed that,

$$A = A^\dagger, B = B^\dagger, R = S^\dagger, \text{ and } R \neq S, \quad \therefore \hat{I}^> \neq \hat{I}^< \quad (13)$$

Thus it is easy to see that we have correspondingly two directions of time $\hat{t}^>$ (for forward motion) and $\hat{t}^<$ (for backward motion) defined by,

$$\hat{t}^> = t \times \hat{I}_t^>, \quad \hat{t}^< = t \times \hat{I}_t^< \quad (14)$$

Correspondingly, the *genoisodual multiplying units* and *forward and backward times* are given by,

$$\hat{I}_t^{d>} = -(\hat{I}_t^>)^\dagger, \quad \hat{t}^{d>} = t \times \hat{I}_t^{d>}, \quad \hat{I}_t^{d<} = -(\hat{I}_t^<)^\dagger, \quad \hat{t}^{d<} = t \times \hat{I}_t^{d<} \quad (15)$$

Thus it is evident that we have following four directions of time using *geno-* and *genoisodual-mathematics* depicted in Figure 3.

GENOTIMES

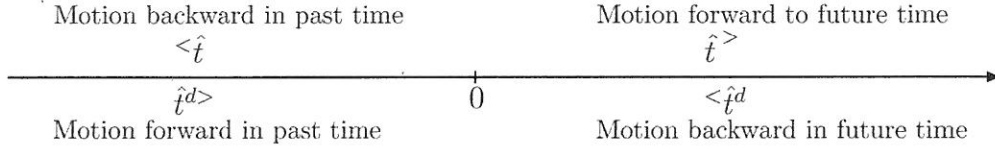


Figure 3: A schematic view of the four possible “Genotime’s directions”. It coincides that of Figure 2 when forward and backward genounits coincide.

As the irreversibility is permitted by the *forward* and *backward genotimes* $\hat{t}^>$ and $\hat{t}^<$ the same gets inherited by the corresponding isodual map namely $\hat{t}^{d>}$ and $\hat{t}^{d<}$ for antimatter systems. However, in biological systems there we have complex irreversible processes occurring simultaneously in that some are structure breaking and the other are structure making. The latter ones get amicably described by isodual genotimes without getting involved the features of antimatter [16].

2.6 Genotopic law of numerical values of time

Herein too for the sake of demonstrative clarity we express the first part of eq.(14) as follows,

$$t_{ext} = t_{ext} \times 1 = \hat{t}_{intr}^> = t_{intr} \times \hat{I}_t^> \quad (16)$$

Thus the pace of t_{intr} would be faster for $\hat{I}_t^> < 1$ and it would be slower for $\hat{I}_t^> > 1$ as compared to our time t_{ext} . The advantage of the genotopic description over the isotopic one is that the irreversibility of the involved processes gets inherently prescribed in the former.

2.7 Directions of time via genohyper- and its isodual-mathematics

By no means *genotheories* are sufficient to represent in entirety intricacies of nature, e.g., because they are unable to represent *biological structures* such as a cell or a seashell. The latter systems are indeed open-nonconservative-irreversible, yet they possess a structure dramatically more complex than that of a nonconservative Newtonian system. A study of the issue has revealed that the limitation of genotheories is due to their single-valued character.

As an illustration, mathematical treatments complemented with computer visualization [16] have established that the shape of seashells can be well described via the conventional single-valued three-dimensional Euclidean space and geometry according to the empirical perception of our three Eustachian tubes. However, the same space and geometry are basically insufficient to represent the growth in time of seashells. In fact, computer visualization shows that, under the exact imposition of the Euclidean axioms, seashells first grow in time in a distorted way and then crack.

Illert [16] showed that a minimally consistent representation of the seashells growth in time requires six dimensions. But seashells exist in our environment and can be observed via our three-dimensional perception. The solution of this apparent dichotomy was proposed [16] via multi-valued hypermathematics essentially characterized by the relaxation of the single-

valued nature of the genounits while preserving their nonsymmetric character (as a necessary condition to represent irreversible events), according to the rules [14, 15]:

$$\hat{I}^> = \left\{ \hat{I}_1^>, \hat{I}_2^>, \hat{I}_3^>, \dots \right\} = 1/\hat{S}, \quad (17)$$

$$\begin{aligned} A > B &= \left\{ A \times \hat{S}_1 \times B, A \times \hat{S}_2 \times B, A \times \hat{S}_3 \times B, \dots \right\}, \\ \hat{I}^> > A &= A > \hat{I}^> = A \times I, \end{aligned} \quad (18)$$

$$<\hat{I} = \left\{ <\hat{I}_1, <\hat{I}_2, <\hat{I}_3, \dots \right\} = 1/\hat{R}, \quad (19)$$

$$\begin{aligned} A < B &= \left\{ A \times \hat{R}_1 \times B, A \times \hat{R}_2 \times B, A \times \hat{R}_3 \times B, \dots \right\}, \\ <\hat{I} < A &= A << \hat{I} = I \times A, \end{aligned} \quad (20)$$

$$A = A^\dagger, B = B^\dagger, \hat{R} = \hat{S}^\dagger \quad (21)$$

All aspects of the bimodular genotheories admit a unique, and significant extension to the above hyperstructures and their explicit form is omitted here for brevity [14, 15]. The expression of the theory via *weak equalities and operations* was first studied by Santilli and Vougiouklis in Ref. [17].

The *isodual hypermathematics* has been developed on the same lines as that in the case of genomathematics. The reader is advised to refer the original literature for example see [3]. Hence corresponding four sets of time's arrows emerge that has been depicted in Figure 4.

These are the *multivalued forward and backward genohypertimes* and are the ultimate generalization of arrow of time. Santilli and Illert have shown [16] that these times help amicably in describing biological growth processes such as the growth of seashells shown in Figure 5.

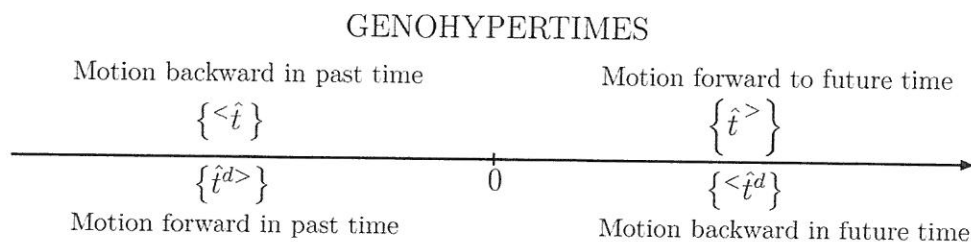


Figure 4: A schematic view of the four sets of possible “Genohypertime’s directions”. It coincides with that of Figure 2 when all forward and backward genhyperounits coincide and the said multivalued character is lost.

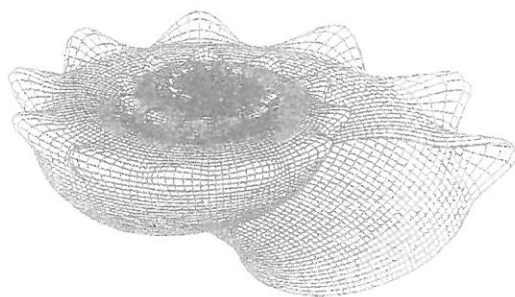


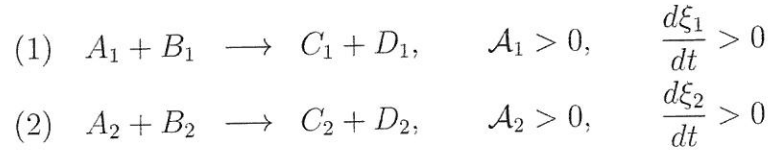
Figure 5: Computer generated Seashell by Santilli and Illert using Hypertimes and Hyperspaces of Santilli [16].

2.8 Four genothermodynamic directions of time

The second law of thermodynamics determines the natural forward direction of processes corresponding to the future direction of time. It has been shown in nonequilibrium thermodynamics that in coupled chemical reactions *one process proceeds in accordance of the dictates of the second law of*

thermodynamics that drives other chemical process to proceed in the direction forbidden by the second law of thermodynamics.

Let us consider two chemical reactions occurring simultaneously in a closed system at constant temperature and pressure.



The nonequilibrium thermodynamic expression of rate of entropy production reads as [18],

$$\frac{d_i S}{dt} = \frac{\mathcal{A}_1}{T} \frac{d\xi_1}{dt} + \frac{\mathcal{A}_2}{T} \frac{d\xi_2}{dt} \geq 0 \quad (22)$$

Now if the chemical reaction 1 proceeds in accordance with the dictates of the second law of thermodynamics we have $\mathcal{A}_1 > 0$ and $\frac{d\xi_1}{dt} > 0$. When it is a case of *coupled chemical reactions* the second chemical reaction proceeds against the dictates of the second law of thermodynamics. Hence, we have $\mathcal{A}_2 < 0$ and $\frac{d\xi_2}{dt} > 0$.

Therefore, we have,

$$\frac{d\xi_2}{dt} > -\frac{\mathcal{A}_1}{\mathcal{A}_2} \frac{d\xi_1}{dt} > 0 \quad (23)$$

Normally it is observed that $-\frac{\mathcal{A}_1}{\mathcal{A}_2} > 1$ and hence there we have $\frac{d\xi_2}{dt} > \frac{d\xi_1}{dt}$. That is *the process that should have not proceeded in the forward direction actually proceeds in that direction that too faster than the process that drives it*.

Now before further carrying out our discussion, at this juncture, we also need to realize that eq.(22) has been expressed using conventional mathematics but as we are dealing with irreversibility of the processes we have to express it using genomathematics that gives the following expression for the *rate of genoentropy production* [5], namely:

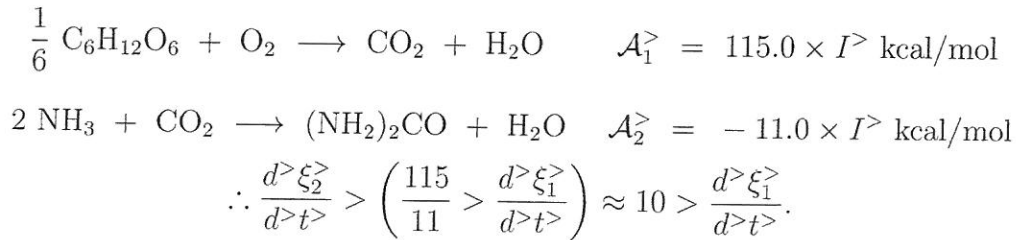
$$\frac{d_i^> S^>}{d^> t^>} = \frac{\mathcal{A}_1^>}{T^>} > \frac{d^> \xi_1^>}{d^> t^>} + \frac{\mathcal{A}_2^>}{T^>} > \frac{d^> \xi_2^>}{d^> t^>} \geq 0^> \quad (24)$$

Thus we see that

1. genothermodynamically allowed conditions are $\mathcal{A}^> > 0^>$ with $\frac{d^>\xi^>}{d^>t^>} > 0^>$ and $\mathcal{A}^> < 0^>$ with $\frac{d^>\xi^>}{d^>t^>} < 0^>$. Thus the former condition describes the *genothermodynamically allowed forward direction* whereas the latter condition describes the *genothermodynamically allowed backward direction*.
2. In genothermodynamic settings the *genothermodynamically unfavourable directions* are determined by $\mathcal{A}^> < 0^>$ with $\frac{d^>\xi^>}{d^>t^>} > 0^>$ and $\mathcal{A}^> > 0^>$ with $\frac{d^>\xi^>}{d^>t^>} < 0^>$.

We recall that the nonequilibrium thermodynamics has produced an explanation of the observation that *in biological systems urea and ATP are synthesized exceedingly efficiently* though their direct synthesis in laboratory could not be achieved. This is so because their synthesis is thermodynamically forbidden. The driving biochemical reactions for urea and ATP syntheses are burning of sucrose and hydrolysis of creatine phosphate respectively [19, 20]. They are depicted below.

(1) The case of biological burning of sugar that couples the production of urea:



(2) The case of biological hydrolysis of Creatine phosphate that couples the ATP (the energy store house of biological systems) synthesis:





$$\therefore \frac{d^>\xi_2^>}{d^>t^>} > \left(\frac{43.5}{39.8} > \frac{d^>\xi_1^>}{d^>t^>} \right) \approx 1.1 > \frac{d^>\xi_1^>}{d^>t^>}$$

From the above we learn that *the forward and backward directions for thermodynamically favourable processes identified by genothermodynamics are distinctly different than those for thermodynamically unfavourable processes*. This then demands the *identification of four thermodynamic directions and corresponding time's arrow*. This fact, has been recently used by the present author [5] by way of *identifying genothermodynamic directions of time based on above described four genotimes discovered for the first time by Santilli*.

1. For the processes that follow the dictates of the second law of thermodynamics we use the geno-space and
2. for the processes that proceed against the dictates of the second law of thermodynamics we use the genoisodual-space.
3. Notice that the use of genoisodual space is without its original purpose of describing antimatter systems.

In this way we are led to distinctly different spaces named as *genothermodynamic space* and *genoisodual-thermodynamic space*. These directions of genothermodynamic times have been depicted in Figure 6. We stress that *one of the advantages of these identified genothermodynamic times is that all processes occurring in the respective spaces* (that is in geno- and genoisodual- thermodynamic spaces) *get described as following the dictates of the second law of thermodynamics as if they are uncoupled processes* [5].

2.9 Genothermotopic law of numerical values of time

Again consider the following identity, namely:

$$t_{ext} = t_{ext} \times 1 = \hat{t}_{geno-therm}^> = t_{geno-therm} \times \hat{I}_{\hat{t}(geno-therm)}^> \quad (25)$$

GENOTHERMODYNAMIC TIMES

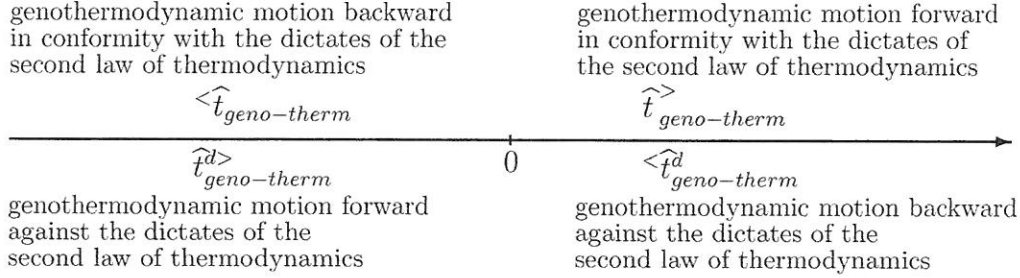


Figure 6: A schematic view of the four sets of possible “Genothermodynamic time’s directions”.

Thus we notice that $t_{geno-therm}$ would have *slower pace* compared to t_{ext} if $\widehat{I}_{i(geno-therm)}^> > 0$. Whereas $t_{geno-therm}$ would have *faster pace* compared to t_{ext} if $\widehat{I}_{i(geno-therm)}^> < 0$. In this way we are at will to choose the value of $\widehat{I}_{i(geno-therm)}^>$ to suit the observations.

Another *superiority of geno-therm times* over the conventional nonequilibrium thermodynamic description via coupled processes is as follows. The deduction based on coupled processes is for example eq.(23). Its only prediction is that *the process that is forbidden to occur by the second law of thermodynamics in reality proceeds much faster than the process that drives it*. However, in the *description using geno-therm space and time* we are led to the exact value of the observed rate of thermodynamically forbidden process.

3 Concluding remarks

The four directions of time’s arrow discovered for the first time by Santilli [1–3] by using his iso-, geno-, hyper- and their isodual- mathematics is a mile stone in the history of science.

Recall that earlier Eddington had asserted that there are two arrows of time directed to forward (future) and backward (past) motions [4]. That

gets augmented to four directions of time with the advent of Santilli's new mathematics. An amazing discovery by Professor Santilli. From the above description it is clear that Eddington's assertions about exclusive bondage of time's arrows and entropy function still remains unaffected. Rather its new facets get surfaced out that is now we have four directions of time an exclusive property of corresponding entropy function.

The advantage of Santilli's this discovery is far reaching. Let us quote what Santilli himself has repeatedly emphasized taking an example of *iso-time* [1-3, 9, 11]:

When we look at a seashell in our hands, we perceive it as evolving according to our time, while in reality the seashell could evolve according to an intrinsic time arbitrarily in the future or the past with respect to our time, the differences depending on the value of the internal isounit of time. The compatibility of different internal times with our own time is assured by the isotopies because the numerical value of the properly formulated isotopic time (that on a time isospace over a time isofield) coincides with our own time. In different words, consider an external entity, such as a seashell, or external event in the universe, such as supernova explosion. Their view we perceive via reflected light, by no means, implies that the considered external entity or event evolves according to our own time, since it could evolve in four completely different, forward and backward, future and or past time. This is a reason for my believing that the raising of questions such as "the age of the universe" is an indication of our ignorance vis-à-vis the complexity of nature because, when we include antimatter, the total time of the universe could well be identically null as a condition to avoid discontinuity at creation.

An another advantage of the discovery by Santilli of four directions of time's arrow that has been derived by the present author [5] through the development of genononequilibrium thermodynamics is the surfacing out of four *genothermodynamic arrows of time* based on the directions of processes dictated by the second law of thermodynamics. The corresponding directions of time are depicted by $\hat{t}_{\text{geno-therm}}^>$, $\hat{t}_{\text{geno-therm}}^d$, $\hat{t}_{\text{geno-therm}}^<$ and $\hat{t}_{\text{geno-therm}}$. These times are to be used

- for forward and backward processes occurring in accordance with the

dictates of the second law of thermodynamics and

- *for the forward and backward processes occurring by flouting the dictates of the second law of thermodynamics as described in the main text.*

One of the notable outcomes of identifying geno- and genoisodual- thermodynamic spaces is that in all the four corresponding spaces the processes occur as following the dictates of the second law of thermodynamics. *Once geno-thermodynamics has provided the permission for a process to occur as described above the next step is to investigate its constitutive description.* This in the case of seashells has already been investigated successfully by Illert and Santilli [16].

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ORIGIN OF QUARKS THROUGH FERMIODYNAMICS

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Abstract

Charge-mass equivalence relations are obtained for bosons and fermions through the equations of continuity from their equations in Generalized Electrodynamics. These relations lead to the m^2 and $m^{3/2}$ laws for the variation of charge with mass of the bosons and fermions respectively. These lead to the quark theory for the constitution of the masses of fermions. Using the $m^{3/2}$ law it is found that there are quarks (u-quarks) of three flavors each of charge $\frac{2}{3}$ and also there are other quarks (d-quarks) of three flavors each of charge $\frac{1}{3}$. It is seen that fermions have three constituent quarks while bosons have two.

Keywords: Fermiodynamics, charge mass relation, quarks, flavors, colors.

1 Introduction

Gell-Mann [1] and Zweig [2] hypothesized fractionally charged particles as constituents of hadrons and bosons. However, it is not known how they evolve in the elementary particle theories *i.e.* it is needed to know theoretically that quarks are constituents of the particle structures and that they possess fractional charges as hypothesized. The fermion equations in Dirac form as developed in the previous paper [3] provide solution to this problem through the equivalence relation between the electromagnetic charge and the particle mass. Similar relation is also obtained for bosons [4], [5] while obtaining boson equations in the Generalized Electrodynamics. We show in the following that there exist fractional charged quarks of six flavors each of three colors as hypothesized. It is also shown that fermions need three quarks as their constituents while bosons need two, one of which is an antiquark.

2 Charge-mass Equivalence Relation

The equation of fermions in Fermiodynamics is [1] given by

$$\hat{\partial}\phi = J, \quad \phi = \hat{\partial}V, \quad \partial^2 V = J \quad (1)$$

where

$$J = iM\phi = \frac{1}{2} [\bar{\phi}\phi + \Gamma^5 (\bar{\phi}\Gamma^5\phi)]\phi \quad (2)$$

The fermion equation is given by

$$(\hat{\partial} + iM)\phi = 0 \quad (3)$$

Similarly, the equation for massive bosons in the Generalized Electrodynamics is given by [4], [5]

$$\partial_\nu G^{\mu\nu} = J^\mu \quad (4)$$

$$G^{\mu\nu} = f_0 g^{\mu\nu} + f^{\mu\nu}$$

$$f^{\nu\mu} = -f^{\mu\nu} \quad (5)$$

$$f^{0i} = f^i, \quad f^{ij} = i\epsilon^{ijk} f_k \quad (6)$$

The source required for massive bosons is given by

$$J^\mu = \mp m^2 V^\mu \quad (7)$$

The boson equation is given by

$$\partial_\nu G^{\mu\nu} \pm m^2 V^\mu = 0 \quad (8)$$

The lower signs in eqns. (7) and (8) are for tachyon bosons [6], [7], [8], [9].

3 Fermion Charge

We estimate from the relation (2) the charge of fermion as (the volume Ω is explicitly introduced)

$$\bar{J}J = \bar{\phi} M^2 \phi = m^3 \cos \theta \Omega^{-3} \quad (9)$$

Now $\bar{J}J = \rho_0^2$ where ρ_0 is Lorentz invariant electromagnetic charge density. By taking electromagnetic total charge as given by

$$q^2 = \rho_0^2 \Omega^2 \quad (10)$$

eqn. (9) gives

$$q^2 = m^3 \cos \theta \Omega^{-1} \quad (11)$$

This is the equivalence relation between the *electromagnetic charge* and the *mass charge* of the particle. It is a $m^{3/2}$ law which we obtained here for the first time.

We observe from eqn. (11) that

1. Massless particles (neutrinos) are electromagnetically neutral.
2. Massive neutral particles have pseudoscalar mass ($\cos \theta = 0$).

The $m^{3/2}$ law is not compatible with the experimental observation because free charged particles possess charge equal to the charge of electron. To reconcile this paradox we introduce the concept of *rate of change of charge*

with respect to mass or the charge to mass rate as $\frac{dq}{dm}$ and this will regulate the growth of the charge with mass. For a given particle, θ and Ω are fixed and so we get on differentiating eqn. (11) w.r.t. mass

$$2q \frac{dq}{dm} = 3m^2 \cos \theta \Omega^{-1} \quad (12)$$

Dividing eqn. (12) by eqn. (11) we get

$$2 \frac{dq}{q} = 3 \frac{dm}{m} \quad (13)$$

This is a linear differential equation between charge and mass. It gives the charge of the particle as

$$q = \frac{2}{3} \frac{dq}{dm} m \quad (14)$$

This equation enables the charge-mass rate $\frac{dq}{dm}$ to regulate the growth of the charge with mass.

4 The Sector Model: The Origin of Quarks

Particles such as hadrons are not point particles, they have mass structure. We, therefore, introduce the *sector model* for them as follows. We divide the mass m into n sectors of masses m_i such that $m = \sum_{i=1}^n m_i$. Each sector has its own charge-mass rate such that for every sector (the index i is not summed here)

$$\left(\frac{dq}{dm} \right)_i m_i = e \quad (15)$$

i.e., each sector has charge equal to e . Then the charge of the particle constituted by such n sectors is given by

$$q = \frac{2}{3} \sum_{i=1}^n \left(\frac{dq}{dm} \right)_i m_i = \frac{2}{3} ne \quad (16)$$

Now particles have charges equal to electron charge or minimum integral multiples of e in the case of resonances. In eqn. (16) the number of sectors needed to have q to be equal to the minimum integral multiple of e is $n = 3$. Such particle has charge $q = 2e$ and each sector contributes a charge $\frac{2}{3}e$. These are the fractionally charged sectors and in particle physics they are known as quarks. According to Pauli's exclusion principle these three sectors or quarks must come in three different kinds called as *flavors* in quark Physics or if they are of the same kind then they must come in three different *colors*. Thus the sectors (quarks) of charge $+\frac{2}{3}e$ are of three flavors each of three colors and these are u (*up*), c (*charm*) and t (*top*) as are popular in particle physics. The quarks of opposite charges are their antiquarks.

The sectors which we have considered above are assumed not to be further divisible. They have therefore, point structure. If the sector which has point structure belongs to the particle as a whole *i.e.* the particle itself has point structure and is a single sector then $n = 1$. According to eqn. (16) the charge of such particle is $q = \frac{2}{3}e$ and it is a u -family quark. Thus the GDM equation (eqn. 3) in Dirac form describes individual quarks also.

Now consider other particle which is divided into n sectors but in which the charge-mass variation rate is now half of that in the u quark. In this case the charge of the particle constituted by such sectors is given by

$$q = \frac{2}{3} \sum_{i=1}^n \left(\frac{dq}{dm} \right)_i m_i = \frac{2}{3} \sum_{i=1}^n \frac{1}{2} \left(\frac{dq}{dm} \right)_{uu} m_i = \frac{1}{3} ne \quad (17)$$

This equation also suggests that for the charge q to be minimum multiples of electron charge e the value of n should be equal to 3. Such particle has charge $q = e$ and each sector contributes a charge $q = \frac{1}{3}e$. These are another variety of fractionally charged sectors (quarks). These are also of three flavors each of three colors and these are known as d (*down*), s (*strange*) and b (*bottom*) quarks. These quarks are also described by eqn. (3). The signs of the charges of the u and d quarks are so selected that they form isodoublets. Thus the isopartners of the u quarks of charge $+\frac{2}{3}e$ are the d quarks of charge $-\frac{1}{3}e$. In

totality, there are quarks of six flavors each of three colors and so 18 in all. There are only six flavors of quarks. This answers the question raised by Kane [10] as to why there are quarks only of six flavors.

The above considerations are made for particles which are made of homogeneous sectors of the same generation. Particles can be made of heterogeneous mixture of sectors of varieties of generations *i.e.* particles can involve combinations of u, d, c, s, t , and b quarks taken three at a time.

5 Charge of Bosons

As in the case of fermions we estimate the charge of the bosons using the charge-mass equivalence relation eqn. (7) as

$$j^{*\mu} j_{\mu} = m^4 V^{*\mu} V_{\mu} \quad (18)$$

Now $j^{*\mu} j_{\mu} = \rho_0^2$ where ρ_0 is Lorentz invariant electromagnetic charge density ($\rho_0^2 = \rho_{m0}^2 + \rho_{e0}^2$). With arguments similar to those above eqn. (10) we get for the boson charge

$$q^2 = m^4 V^{*\mu} V_{\mu} \Omega^{-2} \quad (19)$$

This gives

$$q = m^2 \sqrt{V^{*\mu} V_{\mu}} \Omega^{-1} \quad (19a)$$

Here Ω is the volume of the particle.

This is the m^2 law of variation of boson charge which is different from the $m^{3/2}$ law for fermion charge. For a boson its volume and $V^{*\mu} V_{\mu}$ are constant. Eqn. (19) then gives the following differential equation

$$\frac{dq}{q} = 2 \frac{dm}{m} \quad (20)$$

This gives charge-mass relation as

$$q = \frac{1}{2} \frac{dq}{dm} m \quad (21)$$

5.1 Sector Model for Bosons

Dividing the boson mass into n sectors each of charge e eqn. (21) becomes

$$q = \frac{1}{2} \sum_{i=1}^n \left(\frac{dq}{dm} \right)_i m_i = \frac{1}{2} ne. \quad (22)$$

This equation suggests that the number of sectors needed to have q to be equal to the minimum integral multiple of e is $n = 2$. If q_1 and q_2 are the charges of the contributing sectors eqn. (22) gives

$$\lambda e = q_1 + q_2 \quad (23)$$

where $\lambda = 0, \pm 1$. If $q_1 = q_2$ then each charge is equal to $\pm \frac{1}{2}e$. These are also fractionally charged particles but they are different from the constituent quarks of fermion. However, all the matter of the universe must be created from the *same origin*. Hence the constituent fractionally charged particles of fermions and bosons must be the *same*. The particles of charge $\pm \frac{1}{2}e$ cannot constitute fermions but quarks can constitute bosons. Therefore, the possibility of particles of charges $\pm \frac{1}{2}e$ gets ruled out. But eqn. (23) for bosons is satisfied by quarks only if they come in quark-antiquark pair of similar or dissimilar flavors and colors. This is true for the constitution of spin zero bosons also.

6 Discussion

We briefly list here the important outcomes of the investigations we obtained here.

1. The simple removal of Lorenz conditions on potentials in Dirac-Maxwell electrodynamics [4], [5] led to provide equations for spin-1 bosons and fermions thereby opening two branches of dynamics-Bosodynamics and Fermiodynamics. These dynamics provide equivalence between electromagnetic charges and particle masses *i.e.* the theories treat charges and the masses on equal footing and so are treated the weak, nuclear and the strong (color) fields. The evolution of the dynamics of these fields originates from electrodynamics.
2. The charge-mass relation obeys $m^{3/2}$ law for fermions and the m^2 law for bosons.

3. The charge-mass relation leads to the invention of quarks which came to Particle Physics by way of hypothesis. However, hypothesis does not have proof. If some hypothesis gets proved then it turns into a reality. Thus existence of quarks is a reality and now they are not hypothesis.
4. Three quarks are required for fermions while two are needed for bosons in the form of quark-antiquark pair of similar or dissimilar flavor and color.
5. The theory involves only six flavors of quarks each of three colors. This has answered the curious question of Kane [10] "why there are six quarks only and why not more?"

7 Acknowledgements

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**GRAVITO-ELECTROMAGNETISM EXPLAINED
BY THE THEORY OF INFORMATIONS**

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Abstract

The phenomenon that material bodies can influence each other “at a distance” provokes the idea that they , in one way or another, must exchange data about their presence in spacetime.

In this article we develop this idea: we explain the gravito-electromagnetic description of the gravitational interactions by the hypothesis that “*information*” is the substance of gravitational fields.

The constituent element of that substance is called an “*informaton*”. The theory starts from the idea that any material object manifests itself in space by the emission - at a rate proportional to its rest mass - of informatons: granular mass and energy less entities rushing away with the speed of light and carrying information about the position (“*g-information*”) and about the velocity (“ *β -information*”) of their emitter.

The gravitational field is identified as a cloud of informatons and mathematically described by two vectorial quantities: the “*g-field*” and the “*g-induction*”, respectively characterizing the density of the flow of *g-information* and the density of the cloud of *β -information* at an arbitrary point. The laws of gravito-electromagnetism are mathematically deduced from the kinematics of the informatons; the gravitational interactions are explained as the effect of the tendency of a material object to accelerate in order to become blind for flows of information generated by other objects; and gravitons are identified as informatons carrying a quantum of energy.

Keywords: gravitation, gravitational field, gravito-electromagnetism, informaton, theory of informatons.

INTRODUCTION

Daily contact with the things on hand confronts us with their *substantiality*. An object is not just form, it is also matter. It takes space, it eliminates emptiness. The amount of matter within the contours of a physical body is called its *mass*. The mass of an object manifests itself when it interacts with other objects. A fundamental form of interaction is "*gravitation*". Material objects (masses) attract each other and, if they are free, they move to each other along the straight line joining them.

According to the classical theory of fields, the gravitational interactions can be described by introducing the "*gravitational field*": each material object manifests its substantiality in space by creating and maintaining a vector field and each object in that field experiences a tendency to change its state of motion. The "*field theory*" considers the gravitational field as the entity that mediates in the gravitational interaction. This is further developed by Oliver Heaviside [1] and Oleg Jefimenko [2]: in the "*theory of gravito-electromagnetism*" (GEM) they describe the gravitational field starting from the idea that it must be isomorphic with the electromagnetic one. This implies that it should be characterized by two vectorial quantities that are analogue to respectively the electric field $\vec{E}$ and the magnetic induction $\vec{B}$, and that the relations governing these quantities should be analogue to Maxwell's laws.

Within the framework of general relativity, GEM. has been discussed by a number of authors [3]. It is shown that the gravitational analogues to Maxwell's equations (the GEM equations) can be derived in the weak field approximation [4].

Although GEM describes the gravitational phenomena in a correct and coherent manner, it doesn't create clarity about the physical nature of gravity: the gravitational field is considered as a purely mathematical construction. In this article we develop the idea that, if masses can influence each other "at a distance", they must in one way or another exchange data. We assume that each mass emits information relative to its magnitude and its position, and that it is able to "interpret" the information emitted by its neighbours. In this way we propose a physical

foundation of GEM by introducing "*information*" as the substance of the gravitational field.

We start from the idea that the gravitational field of a material object can be explained as the macroscopic manifestation of the emission by that object - at a rate proportional to its rest mass - of mass and energy less granular entities that - relative to an inertial reference frame - are rushing away with the speed of light and are carrying information about the position ("*g-information*") and the velocity (" *β -information*") of the emitter. Because they transport nothing else than information, we call these entities "*informatons*". In the "*postulate of the emission of informatons*", we define an informaton by its attributes and determine the rules that govern the emission by a point mass that is anchored in an inertial reference frame $\mathbf{O}$. The first consequence of that postulate is that a point mass at rest - and by extension any material object at rest - is the source of an expanding cloud of informatons in $\mathbf{O}$, that - at an arbitrary point P - is characterised by the vectorial quantity $\vec{E}_g$. $\vec{E}_g$ is the density of the flow of *g-information* in that point. That cloud of informatons can be identified with the gravitational field in $\mathbf{O}$ and the quantity $\vec{E}_g$ with the gravitational field strength. A second consequence is that the informatons emitted by a point mass that is moving relative to $\mathbf{O}$, constitute a gravitational field in $\mathbf{O}$ that is characterised by two vectorial quantities: $\vec{E}_g$, the density of the *g-information* flow, and $\vec{B}_g$, the density of the *β -information* cloud. We show that the relations between these two quantities (the laws of GEM) arising from the kinematics of the informatons, are the gravitational analogues of Maxwell's electromagnetic laws. Next we explain the gravitational interaction between masses as the response of a point mass to the disturbance of the symmetry of its "own" gravitational field by the field that, in its direct vicinity, is created and maintained by other masses. And finally we examine the emission of energy by an accelerating mass.

1 THE POSTULATE OF THE EMISSION OF INFORMATONS

1.1 Preliminary definitions

A material body occupies space, its surface encloses matter. The amount of matter within its contours is called its *mass*. According to the field theory, any material body is the source of a gravitational field that at a sufficiently large distance is independent of the form of the body. This "far field" can be calculated by reducing the body to a mathematical point in which all the mass is accumulated. Such a point is called a "*particle*" or a "*point mass*" and it will be graphically represented by a little sphere. If we can calculate the gravitational field generated by a point mass, integral calculus delivers the methods to calculate the gravitational field generated by any material body. This justifies the fact that we in the first instance focus on the emission of informatons by a point mass.

The phenomena that are the subject of this article are situated in spacetime: they are located in "space" and dated in "time".

- In the context of the theory of informatons *space* is conceived as a three-dimensional, homogeneous, isotropic, unlimited and empty continuum. This continuum is called the "Euclidean space" because that what there geometrically is possible is determined by the Euclidean geometry. By anchoring a standardized Cartesian coordinate system to a reference body, an observer can - relative to that reference body - localize each point by three coordinates x, y, z .

- In the same context we identify *time* with the monotonically increasing quantity t that is generated by a standard clock*. In a Cartesian coordinate system a standard clock links to each event a "moment" - this is a specific value of t - and to each duration a "period" or "time interval" - this is a specific increase of t . The introduction of time makes it possible for the observer to express, in an objective manner, the chronological order of events in a Cartesian coordinate system.

* The operation of a standard clock is based on the counting of the successive cycles of a periodic process that is generated by a device inside the clock.

A Cartesian coordinate system together with a standard clock is called a "reference frame". We represent a reference frame as $OXYZ(T)$ or, shortly as O . A reference frame is called an "inertial reference frame" if light propagates rectilinear (in the sense of the Euclidean geometry) with constant speed everywhere in the empty space linked to that frame. This definition implies that the space linked to an inertial reference frame is an homogeneous, isotropic, unlimited and empty continuum in which the Euclidean geometry is valid. A reference frame O' moving relative to an inertial reference frame O is itself also an inertial frame. The coordinates of an event linked to the inertial frames O and O' are related by the Lorentz transformation.

1.2 The concept of gravitational information

Newton's law of universal gravitation [8] may be expressed as follows: *The force between any two particles having masses m_1 and m_2 separated by a distance r is an attraction working along the line joining the particles and has a magnitude*

$$F = G \cdot \frac{m_1 m_2}{r^2}$$

where G is a universal constant having the same value for all pairs of particles.

This law expresses the basic fact of gravitation, namely that two masses are interacting "at-a-distance": they exert forces on one another even though they are not in contact.

According to Newton's law $\vec{F}_B$, the force exerted by a particle A - with mass m_1 - on a particle B - with mass m - is pointing to the position of A and has a magnitude:

$$F_B = \left(G \cdot \frac{m_1}{r^2} \right) m$$

The orientation of this force and the fact that it is directly proportional to the mass of A and inversely proportional to the square of the distance from B to A , imply that particle B must receive *information* about the presence in space of particle A : particle A must send information to B

about its position and about its magnitude. This conclusion is independent of the position and the mass of B ; so we can generalize it and posit that

A particle manifests itself in space by emitting information about its mass and about its position.

We consider that type of information as a substantial element of nature and call it "gravitational information" or "*g-information*". We propose that it is transported by mass and energy less granular entities that rush through space with the speed of light (c). These grains of *g-information* are called *informatons*.

1.3 The postulate of the emission of informatons

A material object manifests its presence in space by continuously emitting informatons. The emission of informatons by a material object anchored in an inertial reference frame O , is governed by the *postulate of the emission of informatons*.

A. *The emission* is governed by the following rules:

1. *The emission is uniform in all directions of space, and the informatons diverge with the speed of light ($c = 3 \cdot 10^8$ m/s) along radial trajectories relative to the position of the emitter.*

2. $\dot{N} = \frac{dN}{dt}$, *the rate at which a point-mass emits informatons**, is time independent and proportional to its rest mass m . So, there is a constant K so that:

$$\dot{N} = K \cdot m$$

* We neglect the possible stochastic nature of the emission, that is responsible for noise on the quantities that characterize the gravitational field. So, $\dot{N}$ is the average emission rate.

3. The constant K is equal to the ratio of the square of the speed of light (c) to the Planck constant (h):

$$K = \frac{c^2}{h} = 1,36.10^{50} \text{ kg}^{-1} \cdot \text{s}^{-1}$$

B. We call the essential attribute of an informaton its *g-index*. The g-index of an informaton refers to information about the position of its emitter and equals the *elementary quantity of g-information*. It is represented by a vectorial quantity $\vec{s}_g$:

1. $\vec{s}_g$ points to the position of the emitter.

2. The elementary quantity of g-information is:

$$s_g = \frac{1}{K \eta_0} = 6,18.10^{-60} \text{ m}^3 \cdot \text{s}^{-1}$$

where $\eta_0 = \frac{1}{4\pi G} = 1,19.10^9 \text{ kg} \cdot \text{s}^2 \cdot \text{m}^{-3}$, with G the gravitational constant.

So, according to the *postulate of the emission of informatons*, a particle is an emitter of informatons. The emission rate only depends on its mass m and the fundamental attribute of an informaton is its g-index.

One can imagine an informaton as a quasi infinitely small sphere that is spinning about an axis. $\vec{s}_g$ is along that axis and the orientation of this vector is linked to the rotation of the sphere by the rule of the corkscrew.

Informatons are grains of g-information and, as such, the constituent elements of gravitational fields. In what follows we will show that they macroscopically manifest themselves in the vectorial quantities $\vec{E}_g$ and $\vec{B}_g$, that mathematically characterize those fields, and in the laws of GEM that are manifestations of their kinematics. We will also show that informatons emitted by an accelerated point mass can be carriers of a quantum of energy: in that case they appear to the observer as "gravitons". It also is possible to explain electromagnetism by the theory

of informatons. In that context they macroscopically manifest themselves as $\vec{E}$ and $\vec{B}$, the vectorial quantities that characterize an EM field, and in Maxwell's laws that are manifestations of their kinematics. In the context of EM single informatons can be observed as "photons", as carriers of a quantum of energy emitted by an accelerated point charge.

2 THE GRAVITATIONAL FIELD OF MASSES AT REST

2.1 The gravitational field of a point mass at rest

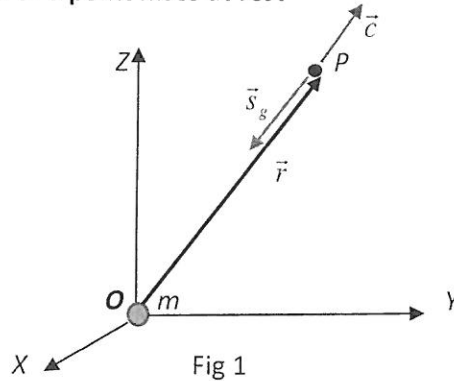


Fig 1

In fig 1 we consider a point mass that is anchored in the origin of an inertial reference frame O . According to the postulate it continuously emits informatons in all directions of space.

The informatons that with velocity $\vec{c} = c \cdot \frac{\vec{r}}{r} = c \cdot \vec{e}_r$ pass near a fixed point P - defined by the position vector $\vec{r}$ - are characterised by their g-index $\vec{s}_g$:

$$\vec{s}_g = -\frac{1}{K \cdot \eta_0} \cdot \frac{\vec{r}}{r} = -\frac{1}{K \cdot \eta_0} \cdot \vec{e}_r$$

The rate at which the point mass emits g-information is the product of the rate at which it emits informatons with the elementary g-information quantity:

$$\dot{N} \cdot S_g = \frac{m}{\eta_0}$$

Of course, this is also the rate at which it sends g-information through any closed surface that spans m .

The emission of informatons fills the space around m with an expanding cloud of g-information. This cloud has the shape of a sphere whose surface goes away from the centre O - the position of the point mass - with the speed of light.

- Within the cloud there is a *stationary state*: because the inflow equals the outflow, each spatial region contains an unchanging number of informatons and thus a constant quantity of g-information. Moreover, the orientation of the g-indices of the informatons passing near a fixed point is always the same.
- The cloud can be identified with a *continuum*: each spatial region contains a very large number of informatons: the g-information is like continuously spread over the volume of the region.

That cloud of g-information surrounding O constitutes the gravitational field or the g-field of the point mass m .

Without interruption "countless" informatons are rushing through any - even a very small - surface in the gravitational field: we can describe the motion of g-information through a surface as a *continuous flow* of g-information. We know already that the intensity of the flow of g-information through a closed surface that spans O is expressed as:

$$\dot{N} \cdot S_g = \frac{m}{\eta_0}$$

If the closed surface is a sphere with radius r , the *intensity of the flow per unit area* is given by:

$$\frac{m}{4 \cdot \pi \cdot r^2 \cdot \eta_0}$$

This is the *density* of the flow of g-information at each point P at a distance r from m (fig 1). This quantity is, together with the orientation

of the g -indices of the informatons that are passing near P , characteristic for the gravitational field at that point. Thus, at a point P , the gravitational field of the point mass m is defined by the vectorial quantity $\vec{E}_g$:

$$\vec{E}_g = \frac{\dot{N}}{4\pi \cdot r^2} \cdot \vec{s}_g = -\frac{m}{4\pi \eta_0 \cdot r^2} \cdot \vec{e}_r = -\frac{m}{4\pi \eta_0 \cdot r^3} \cdot \vec{r}$$

This quantity is the *gravitational field strength* or the *g-field strength* or the *g-field*. In any point of the gravitational field of the point mass m , the orientation of $\vec{E}_g$ corresponds to the orientation of the g -indices of the informatons which are passing near that point. And the magnitude of $\vec{E}_g$ is the *density of the g-information flow* at that point. Let us note that $\vec{E}_g$ is opposite to the sense of movement of the informatons.

Let us consider a surface-element dS at P (fig 2,a). Its orientation and magnitude are completely determined by the surface-vector $\vec{dS}$ (fig 2,b). By $-d\Phi_G$, we represent the rate at which g -information flows through dS in the sense of the positive normal $\vec{e}_n$ and we call the scalar quantity $d\Phi_G$ the *elementary g-flux through dS* :

$$d\Phi_G = \vec{E}_g \cdot \vec{dS} = E_g \cdot dS \cdot \cos \alpha$$

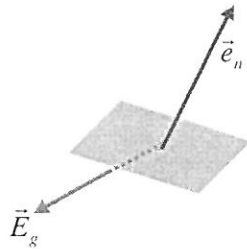


Fig 2,a

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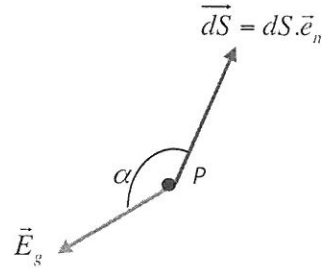


Fig 2,b

For an arbitrary closed surface S that spans m , the outward flux (which we obtain by integrating the elementary contributions $d\Phi_g$ over S) must be equal to the rate at which the mass emits g-information. Thus:

$$\Phi_G = \oint \vec{E}_g \cdot \vec{dS} = -\frac{m}{\eta_0}$$

This relation expresses the conservation of g-information in the case of a point mass at rest.

2.2 The gravitational field of a set of point-masses at rest

We consider a set of point-masses $m_1, \dots, m_i, \dots, m_n$ that are anchored in an inertial frame O . In an arbitrary point P , the flows of g-information who are emitted by the distinct masses are defined by the gravitational fields $\vec{E}_{g1}, \dots, \vec{E}_{gi}, \dots, \vec{E}_{gn}$. $-d\Phi_g$, the rate at which g-information flows through a surface-element dS at P in the sense of the positive normal, is the sum of the contributions of the distinct masses:

$$-d\Phi_G = \sum_{i=1}^n -(\vec{E}_{gi} \cdot \vec{dS}) = -(\sum_{i=1}^n \vec{E}_{gi}) \cdot \vec{dS} = -\vec{E}_g \cdot \vec{dS}$$

So, the *effective density of the flow of g-information at P* (the effective g-field) is completely defined by:

$$\vec{E}_g = \sum_{i=1}^n \vec{E}_{gi}$$

We conclude: *In a point of space, the g-field of a set of point masses at rest is completely defined by the vectorial sum of the g-fields caused by the distinct masses.* Let us note that the orientation of the effective g-field has no longer a relation with the direction in which the passing informations are moving.

One shows easily that the outward g-flux through a closed surface in the g-field of a set of anchored point masses only depends on the spanned masses m_{in} :

$$-\oint_S \vec{E}_g \cdot d\vec{S} = \frac{m_{in}}{\eta_0}$$

This relation expresses the conservation of g-information in the case of a set of point masses at rest.

2.3 The gravitational field of a mass continuum at rest

We call an object in which the matter in a time independent manner is spread over the occupied volume, a *mass continuum*. At each point Q of such a continuum, the accumulation of mass is defined by the (*mass*) *density* ρ_G . To define this scalar quantity one considers the mass dm of a volume element dV that contains Q . The accumulation of mass in the vicinity of Q is defined by: $\rho_G = \frac{dm}{dV}$

A mass continuum - anchored in an inertial frame - is equivalent to a set of infinitely many infinitesimal small mass elements dm . The contribution of each of them to the field strength at an arbitrary point P is $d\vec{E}_g$. $\vec{E}_g$, the effective g-field at P , is the result of the integration over the volume of the continuum of all these contributions.

It is evident that the outward g-flux through a closed surface S only depends on the mass enclosed by the surface (the enclosed volume is V):

$$-\oint_S \vec{E}_g \cdot d\vec{S} = \frac{1}{\eta_0} \cdot \iiint_V \rho_G \cdot dV$$

That relation is equivalent with (theorem of Ostrogradsky [5]):

$$\text{div} \vec{E}_g = -\frac{\rho_G}{\eta_0}$$

This is the expression of the conservation of g-information in the case of a mass continuum at rest.

Furthermore, one can show that [7]: $\text{rot} \vec{E}_g = 0$, what implies the existence of a gravitational potential function V_g for which:

$$\vec{E}_g = -\text{grad} V_g.$$

3 THE GRAVITATIONAL FIELD OF MOVING MASSES

3.1 Rest mass and relativistic mass

In fig 3, we consider a point mass that is moving with constant velocity $\vec{v} = v\vec{e}_z$ along the Z-axis of an inertial reference frame O . At the moment $t = 0$, it passes through the origin O and at $t = t$ through the point P_1 .

We posit that $\dot{N}$ - the rate at which a point mass emits informatons in the space linked to O - is determined by its rest mass m_0 and is independent of its motion: $\dot{N} = \frac{dN}{dt} = K.m_0$

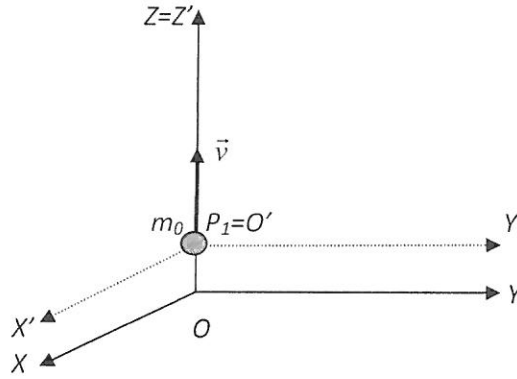


Fig 3

That implies that, if the time is read on a standard clock anchored in O , dN - the number of informatons that during the interval dt by a (whether or not moving) point mass is emitted in the space connected to O - is:

$$dN = K.m_0.dt$$

We also can link the spacetime to an inertial reference frame O' (fig 3) whose origin is anchored to the point mass and that is moving relative to O with the velocity $\vec{v} = v\vec{e}_z$. We assume that $t = t' = 0$ when the mass

passes through O (t is the time read on a standard clock in O and t' the time read on a standard clock in O'). We determine the time that expires while the moving point mass emits dN informatons in the space linked to O .

1. An observer in O uses therefore a standard clock that is linked to that reference frame. The emission of dN informatons takes dt seconds. The relationship between dN and dt is:

$$dN = K m_0 . dt$$

2. For an observer in O' , who reads the time on a standard clock linked to this inertial reference frame, the corresponding time interval is dt' .

According to the Lorentz transformation equations [6], the relationship between dt and dt' is:

$$dt = \frac{dt'}{\sqrt{1 - \beta^2}} \quad \text{with} \quad \beta = \frac{v}{c}$$

$$\text{So:} \quad dN = K m_0 . dt = K m_0 . \frac{dt'}{\sqrt{1 - \beta^2}} = K . \frac{m_0}{\sqrt{1 - \beta^2}} . dt' = \frac{\dot{N}}{\sqrt{1 - \beta^2}} . dt'$$

$$\text{and:} \quad \dot{N}' = \frac{dN}{dt'} = \frac{\dot{N}}{\sqrt{1 - \beta^2}} = K . \frac{m_0}{\sqrt{1 - \beta^2}} = K . m$$

with $m = \frac{m_0}{\sqrt{1 - \beta^2}}$, the "relativistic mass"

We conclude: *For an observer in O , the rate at which a moving point mass emits informatons is determined by its rest mass when the emission is considered relative to O , and by the relativistic mass when it is considered relative to O' .*

3.2 The field caused by a uniform rectilinear moving point mass

In fig 4,a, we consider again a point mass with rest mass m_0 that, with constant velocity $\vec{v} = v\vec{e}_z$, moves along the Z -axis of an inertial reference frame O . At the moment $t = 0$, it passes through the origin O and at the moment $t = t$ through the point P_1 . It is evident that:

$$OP_1 = z_{P_1} = vt$$

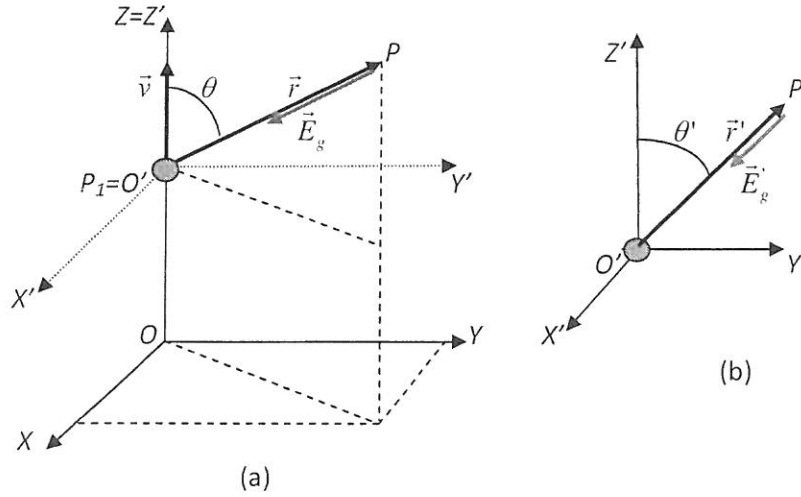


Fig 4

Relative to O' , the instantaneous value of the density of the flow of g-information at P is determined by:

$$\vec{E}_g' = -\frac{m_0}{4\pi\eta_0 r'^3 \sqrt{1-\beta^2}} \vec{r}'$$

Indeed, relative to O' the point mass is at rest and the position of P is determined by the time dependant position vector $\vec{r}'$ or by the Cartesian coordinates (x', y', z') . So, the g-field generated by the mass is determined by §2.1 and it is, according to §3.1, the relativistic mass that determines the emission rate.

The components of $\vec{E}_g$ in $O'X'Y'Z'$, namely:

$$E'_{gx'} = -\frac{m_0}{4\pi\eta_0 r'^3 \sqrt{1-\beta^2}} \cdot x'$$

$$E'_{gy'} = -\frac{m_0}{4\pi\eta_0 r'^3 \sqrt{1-\beta^2}} \cdot y'$$

$$E'_{gz'} = -\frac{m_0}{4\pi\eta_0 r'^3 \sqrt{1-\beta^2}} \cdot z'$$

determine at P the densities of the flows of g-information respectively through a surface element $dy'.dz'$ perpendicular to the X' -axis, through a surface element $dz'.dx'$ perpendicular to the Y' -axis and through a surface element $dx'.dy'$ perpendicular to the Z' -axis. In the time interval dt' , the quantities of g-information flowing through these different surface elements at P are:

$$E'_{gx'} \cdot dy' \cdot dz' \cdot dt' = -\frac{m_0 \cdot x'}{4\pi\eta_0 r'^3 \sqrt{1-\beta^2}} \cdot dy' \cdot dz' \cdot dt'$$

$$E'_{gy'} \cdot dz' \cdot dx' \cdot dt' = -\frac{m_0 \cdot y'}{4\pi\eta_0 r'^3 \sqrt{1-\beta^2}} \cdot dz' \cdot dx' \cdot dt'$$

$$E'_{gz'} \cdot dx' \cdot dy' \cdot dt' = -\frac{m_0 \cdot z'}{4\pi\eta_0 r'^3 \sqrt{1-\beta^2}} \cdot dx' \cdot dy' \cdot dt'$$

The Cartesian coordinates of P in the frames O and O' are connected by [6]:

$$x' = x \quad y' = y \quad z' = \frac{z - vt}{\sqrt{1-\beta^2}} = \frac{z - z_{p_1}}{\sqrt{1-\beta^2}}$$

$$\text{The line elements by: } dx' = dx \quad dy' = dy \quad dz' = \frac{dz}{\sqrt{1-\beta^2}}$$

$$\text{And the time intervals by: } dt' = dt \cdot \sqrt{1-\beta^2}$$

Further: $r' = r \cdot \frac{\sqrt{1 - \beta^2 \cdot \sin^2 \theta}}{\sqrt{1 - \beta^2}}.$

Indeed in O :

$$r = \sqrt{x^2 + y^2 + (z - z_{P_1})^2}, \quad \sin \theta = \frac{\sqrt{x^2 + y^2}}{r} \quad \text{and} \quad \cos \theta = \frac{z - z_{P_1}}{r}$$

and in O' : $r' = \sqrt{x'^2 + y'^2 + z'^2} \quad \text{and} \quad \sin \theta' = \frac{\sqrt{x'^2 + y'^2}}{r'}.$

Expressing r' in function of x, y and z we finally obtain:

$$\begin{aligned} r' &= \sqrt{x^2 + y^2 + \frac{(z - z_{P_1})^2}{(1 - \beta^2)}} = \frac{\sqrt{r^2 \cdot \sin^2 \theta \cdot (1 - \beta^2) + r^2 \cdot \cos^2 \theta}}{\sqrt{1 - \beta^2}} \\ &= r \cdot \frac{\sqrt{1 - \beta^2 \cdot \sin^2 \theta}}{\sqrt{1 - \beta^2}} \end{aligned}$$

So relative to O , the quantities of g-information send by the moving mass - in the positive direction - through the surface elements $dy.dz, dz.dx$ and $dx.dy$ at P are:

$$\begin{aligned} & - \frac{m_0}{4\pi\eta_0 r^3} \cdot \frac{1 - \beta^2}{(1 - \beta^2 \cdot \sin^2 \theta)^{\frac{3}{2}}} \cdot x \cdot dy \cdot dz \cdot dt \\ & - \frac{m_0}{4\pi\eta_0 r^3} \cdot \frac{1 - \beta^2}{(1 - \beta^2 \cdot \sin^2 \theta)^{\frac{3}{2}}} \cdot y \cdot dz \cdot dx \cdot dt \\ & - \frac{m_0}{4\pi\eta_0 r^3} \cdot \frac{1 - \beta^2}{(1 - \beta^2 \cdot \sin^2 \theta)^{\frac{3}{2}}} \cdot (z - z_{P_1}) \cdot dx \cdot dy \cdot dt \end{aligned}$$

Since the densities at P of the flows of g-information in the direction of the X-, the Y- and the Z-axis are the components of the g-field caused by the moving point mass m_0 in P , we find:

$$E_{gx} = -\frac{m_0}{4\pi\eta_0 r^3} \cdot \frac{1 - \beta^2}{(1 - \beta^2 \sin^2 \theta)^{\frac{3}{2}}} \cdot x$$

$$E_{gy} = -\frac{m_0}{4\pi\eta_0 r^3} \cdot \frac{1 - \beta^2}{(1 - \beta^2 \sin^2 \theta)^{\frac{3}{2}}} \cdot y$$

$$E_{gz} = -\frac{m_0}{4\pi\eta_0 r^3} \cdot \frac{1 - \beta^2}{(1 - \beta^2 \sin^2 \theta)^{\frac{3}{2}}} \cdot (z - z_{P_1})$$

So, the g-field caused by the moving point mass at the fixed point P is:

$$\vec{E}_g = -\frac{m_0}{4\pi\eta_0 r^3} \cdot \frac{1 - \beta^2}{(1 - \beta^2 \sin^2 \theta)^{\frac{3}{2}}} \cdot \vec{r} = -\frac{m_0}{4\pi\eta_0 r^2} \cdot \frac{1 - \beta^2}{(1 - \beta^2 \sin^2 \theta)^{\frac{3}{2}}} \cdot \vec{e}_r$$

We conclude:

A point mass describing relative to an inertial reference frame O a uniform rectilinear movement, creates in the space linked to that frame a time dependent gravitational field. $\vec{E}_g$, the g-field at an arbitrary point P , points at any time to the position of the mass at that moment and its magnitude is:*

$$E_g = \frac{m_0}{4\pi\eta_0 r^2} \cdot \frac{1 - \beta^2}{(1 - \beta^2 \sin^2 \theta)^{\frac{3}{2}}}$$

* The orientation of the field strength implies that the spin indices of the informations that at a certain moment pass near P , point to the position of the emitting mass at that moment.

If the speed of the mass is much smaller than the speed of light, this expression reduces itself to that valid in the case of a mass at rest. This non-relativistic result could directly be obtained if one assumes that the displacement of the point mass during the time interval that the informatons need to move from the emitter to P can be neglected compared to the distance they travel during that period.

3.3 The emission of informatons by a point mass describing a uniform rectilinear motion

In fig 5 we consider a point mass m_0 that moves with a constant velocity $\vec{v}$ along the Z -axis of an inertial reference frame O . Its instantaneous position (at the arbitrary moment t) is P_1 . The position of P , an arbitrary fixed point in space, is defined by the vector $\vec{r} = \overrightarrow{P_1P}$. This position vector $\vec{r}$ - just like the distance r and the angle θ - is time dependent because the position of P_1 is constantly changing.

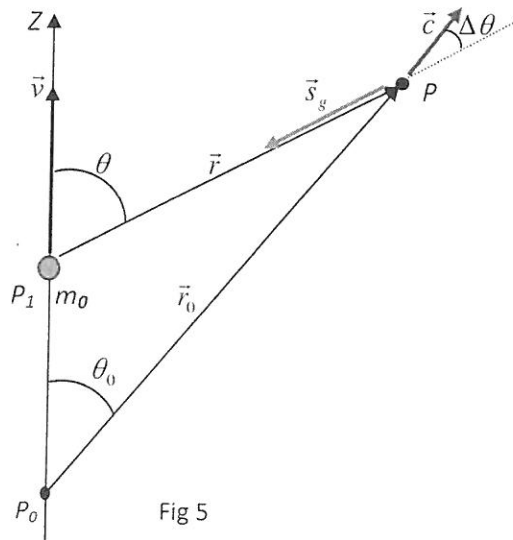


Fig 5

The informatons that - with the speed of light - at the moment t are passing near P , are emitted when m_0 was at P_0 . Bridging the distance

$$P_0P = r_0 \text{ took the time interval } \Delta t = \frac{r_0}{c}.$$

During their rush from P_0 to P , the mass moved from P_0 to P_1 : $P_0P_1 = v \cdot \Delta t$

- $\vec{c}$, the velocity of the informatons, points in the direction of their movement, thus along the radius P_0P .
- $\vec{s}_g$, their g-index, points to P_1 , the position of m_0 at the moment t . This is an implication of rule B.1 of the postulate of the emission of informatons and confirmed by the conclusion of §3.2.

The lines carrying $\vec{s}_g$ and $\vec{c}$ form an angle $\Delta\theta$. We call this angle - that is characteristic for the speed of the point mass - the "*characteristic angle*" or the "*characteristic deviation*". The quantity $s_\beta = s_g \cdot \sin(\Delta\theta)$, referring to the speed of its emitter, is called the "*characteristic g-information*" or the " *β -information*" of an informaton.

We conclude that an informaton emitted by a moving point mass, transports information referring to the velocity of that mass. This information is represented by its "*gravitational characteristic vector*" or " *β -index*" $\vec{s}_\beta$ that is defined by:

$$\vec{s}_\beta = \frac{\vec{c} \times \vec{s}_g}{c}$$

- The β -index is perpendicular to the plane formed by the path of the informaton and the straight line that carries the g-spin vector, thus perpendicular to the plane formed by the point P and the path of the emitter.
- Its orientation relative to that plane is defined by the "rule of the corkscrew".

- Its magnitude is: $s_\beta = s_g \cdot \sin(\Delta\theta)$, the β -information of the informaton.

In the case of fig 5, the β -indices have the orientation of the positive X -axis.

Applying the sine-rule to the triangle P_0P_1P , we obtain:

$$\frac{\sin(\Delta\theta)}{v \cdot \Delta t} = \frac{\sin \theta}{c \cdot \Delta t}$$

It follows:
$$s_\beta = s_g \cdot \frac{v}{c} \cdot \sin \theta = s_g \cdot \beta \cdot \sin \theta = s_g \cdot \beta_\perp$$

$\beta_\perp$ is the component of the dimensionless velocity $\vec{\beta} = \frac{\vec{v}}{c}$ perpendicular to $\vec{s}_g$.

Taking into account the orientation of the different vectors, the β -index of an informaton emitted by a point mass moving with constant velocity, can also be expressed as:

$$\vec{s}_\beta = \frac{\vec{v} \times \vec{s}_g}{c}$$

3.4 The gravitational induction of a point mass describing a uniform rectilinear motion

We consider again the situation of fig 5. All informatons in dV - the volume element at P - carry both g -information and β -information. The β -information refers to the velocity of the emitting mass and is represented by the β -indices $\vec{s}_\beta$:

$$\vec{s}_\beta = \frac{\vec{c} \times \vec{s}_g}{c} = \frac{\vec{v} \times \vec{s}_g}{c}$$

If n is the density at P of the cloud of informatons (number of informatons per unit volume) at the moment t , the amount of β -information in dV is determined by the magnitude of the vector:

$$n \cdot \vec{s}_\beta \cdot dV = n \cdot \frac{\vec{c} \times \vec{s}_g}{c} \cdot dV = n \cdot \frac{\vec{v} \times \vec{s}_g}{c} \cdot dV$$

And the density of the β -information (characteristic information per unit volume) at P is determined by:

$$n \cdot \vec{s}_\beta = n \cdot \frac{\vec{c} \times \vec{s}_g}{c} = n \cdot \frac{\vec{v} \times \vec{s}_g}{c}$$

We call this (time dependent) vectorial quantity - that will be represented by $\vec{B}_g$ - the "gravitational induction" or the "g-induction"* at P :

- Its magnitude B_g determines the density of the β -information at P .
- Its orientation determines the orientation of the β -indices $\vec{s}_\beta$ of the informatons passing near that point.

So, the g-induction caused by the moving mass m_0 (fig 5) at P is:

$$\vec{B}_g = n \cdot \frac{\vec{v} \times \vec{s}_g}{c} = \frac{\vec{v}}{c} \times (n \cdot \vec{s}_g)$$

N - the density of the flow of informatons at P (the rate per unit area at which the informatons cross an elementary surface perpendicular to the direction of movement) - and n - the density of the cloud of informatons at P (number of informatons per unit volume) - are connected by the relation:

$$n = \frac{N}{c}$$

With $\vec{E}_g = N \cdot \vec{s}_g$, we can express the gravitational induction at P as:

$$\vec{B}_g = \frac{\vec{v}}{c^2} \times (N \cdot \vec{s}_g) = \frac{\vec{v} \times \vec{E}_g}{c^2}$$

* This quantity is also called the "cogravitational field", represented as $\vec{K}$ or the "gyrotation", represented as $\vec{\Omega}$.

Taking into account (§3.2): $\vec{E}_g = -\frac{m_0}{4\pi\eta_0 r^3} \cdot \frac{1-\beta^2}{(1-\beta^2 \sin^2 \theta)^{\frac{3}{2}}} \cdot \vec{r}$

We find: $\vec{B}_g = -\frac{m_0}{4\pi\eta_0 c^2 r^3} \cdot \frac{1-\beta^2}{(1-\beta^2 \sin^2 \theta)^{\frac{3}{2}}} \cdot (\vec{v} \times \vec{r})$

We define the constant $\nu_0 = 9,34 \cdot 10^{-27} \text{ m.kg}^{-1}$ as: $\nu_0 = \frac{1}{c^2 \eta_0}$

And finally, we obtain:

$$\vec{B}_g = \frac{\nu_0 \cdot m_0}{4\pi r^3} \cdot \frac{1-\beta^2}{(1-\beta^2 \sin^2 \theta)^{\frac{3}{2}}} \cdot (\vec{r} \times \vec{v})$$

$\vec{B}_g$ at P is perpendicular to the plane formed by P and the path of the point mass; its orientation is defined by the rule of the corkscrew; and its magnitude is: $B_g = \frac{\nu_0 \cdot m_0}{4\pi r^2} \cdot \frac{1-\beta^2}{(1-\beta^2 \sin^2 \theta)^{\frac{3}{2}}} \cdot v \cdot \sin \theta$

If the speed of the mass is much smaller than the speed of light, this expression reduces itself to:

$$\vec{B}_g = \frac{\nu_0 \cdot m_0}{4\pi r^3} \cdot (\vec{r} \times \vec{v})$$

This non-relativistic result could be obtained directly if one assumes that the displacement of the point mass during the time interval that the informations need to move from the emitter to P can be neglected compared to the distance they travel during that period.

3.5 The gravitational field of a point mass describing a uniform rectilinear motion

A point mass m_0 , moving with constant velocity $\vec{v} = v \cdot \vec{e}_z$ along the Z -axis of an inertial frame, creates and maintains an expanding cloud of

informatons that are carrying both g - and β -information. That cloud can be identified with a time dependent continuum. That continuum is called the *gravitational field** of the point mass. It is characterized by two time dependent vectorial quantities: the gravitational field (short: *g-field*) $\vec{E}_g$ and the gravitational induction (short: *g-induction*) $\vec{B}_g$.

- With N the density of the flow of informatons at P (the rate per unit area at which the informatons cross an elementary surface perpendicular to the direction of movement), the g -field at that point is:

$$\vec{E}_g = N \cdot \vec{s}_g = -\frac{m_0}{4\pi\eta_0 r^3} \cdot \frac{1 - \beta^2}{(1 - \beta^2 \cdot \sin^2 \theta)^{\frac{3}{2}}} \cdot \vec{r}$$

The orientation of $\vec{E}_g$ learns that the direction of the flow of g -information at P is not the same as the direction of the flow of informatons.

- With n , the density of the cloud of informatons at P (number of informatons per unit volume), the g -induction at that point is:

$$\vec{B}_g = n \cdot \vec{s}_\beta = \frac{v_0 m_0}{4\pi r^3} \cdot \frac{1 - \beta^2}{(1 - \beta^2 \cdot \sin^2 \theta)^{\frac{3}{2}}} \cdot (\vec{r} \times \vec{v})$$

One can verify that:

$$\begin{array}{ll} 1. \operatorname{div} \vec{E}_g = 0 & 3. \operatorname{rot} \vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t} \\ 2. \operatorname{div} \vec{B}_g = 0 & 4. \operatorname{rot} \vec{B}_g = \frac{1}{c^2} \cdot \frac{\partial \vec{E}_g}{\partial t} \end{array}$$

* Also called: "gravito-electromagnetic" (GEM field) or "gravito-magnetic" field (GM field)

These relations are the laws of GEM in the case of the gravitational field of a point mass describing a uniform rectilinear motion.

If $v \ll c$, the expressions for the g-field and the g-induction reduce to:

$$\vec{E}_g = -\frac{m_0}{4\pi\eta_0 r^3} \cdot \vec{r} \quad \text{and} \quad \vec{B}_g = \frac{v_0 m_0}{4\pi c^3} \cdot (\vec{r} \times \vec{v})$$

3.6 The gravitational field of a set of point masses describing uniform rectilinear motions

We consider a set of point masses $m_1, \dots, m_i, \dots, m_n$ that move with constant velocities $\vec{v}_1, \dots, \vec{v}_i, \dots, \vec{v}_n$ relative to an inertial reference frame O . This set creates and maintains a gravitational field that in each point of the space linked to O , is characterised by the vector pair $(\vec{E}_g, \vec{B}_g)$.

- Each mass m_i continuously emits g-information and contributes with an amount $\vec{E}_{gi}$ to the g-field at an arbitrary point P . As in §2.2 we conclude that the effective g-field $\vec{E}_g$ at P is defined as:

$$\vec{E}_g = \sum \vec{E}_{gi}$$

- If it is moving, each mass m_i emits also β -information, contributing to the g-induction at P with an amount $\vec{B}_{gi}$. It is evident that the β -information in the volume element dV at P at each moment t is expressed by:

$$\sum (\vec{B}_{gi} \cdot dV) = (\sum \vec{B}_{gi}) \cdot dV$$

Thus, the effective g-induction $\vec{B}_g$ at P is:

$$\vec{B}_g = \sum \vec{B}_{gi}$$

On the basis of the superposition principle we can conclude that the laws of GEM mentioned in the previous section remain valid for the effective g-field and g-induction in the case of the gravitational field of a set of point masses describing a uniform rectilinear motion.

3.7 The gravitational field of a stationary mass flow

The term "stationary mass flow" indicates the movement of an homogeneous and incompressible fluid that, in an invariable way, flows relative to an inertial reference frame. The intensity of the flow at an arbitrary point P is characterized by the flow density $\vec{J}_G$. The magnitude of this vectorial quantity equals the rate per unit area at which the mass flows through a surface element that is perpendicular to the flow at P . The orientation of $\vec{J}_G$ corresponds to the direction of that flow. If $\vec{v}$ is the velocity of the mass element $\rho_G dV$ that at the moment t flows through P , then: $\vec{J}_G = \rho_G \cdot \vec{v}$

So, the rate at which the flow transports - in the positive sense (defined by the orientation of the surface vectors $\vec{dS}$) - mass through an arbitrary surface ΔS , is: $i_G = \iint_{\Delta S} \vec{J}_G \cdot \vec{dS}$. We call i_G the *intensity of the mass flow through ΔS* .

Since a stationary mass flow is the macroscopic manifestation of moving mass elements $\rho_G dV$, it creates and maintains a gravitational field. And since the velocity $\vec{v}$ of the mass element in each point is time independent, *the gravitational field of a stationary mass flow will be time independent*. It is evident that the rules of §2.3 also apply for this time independent g-field:

$$\text{div} \vec{E}_g = -\frac{\rho_G}{\eta_0} \quad \text{and} \quad \text{rot} \vec{E}_g = 0 \quad \text{what implies: } \vec{E}_g = -\text{grad} V_g$$

One can prove [7] that the rules for the time independent g-induction are:

$$\text{div} \vec{B}_g = 0 \quad \text{what implies} \quad \vec{B}_g = \text{rot} \vec{A}_g \quad \text{and} \quad \text{rot} \vec{B}_g = -v_0 \cdot \vec{J}_G$$

These are the laws of GEM in the case of the gravitational field of a stationary mass flow.

3.8 The gravitational field of an accelerated point mass.

3.8.1 The g-spinvector of an informaton emitted by an accelerated point mass

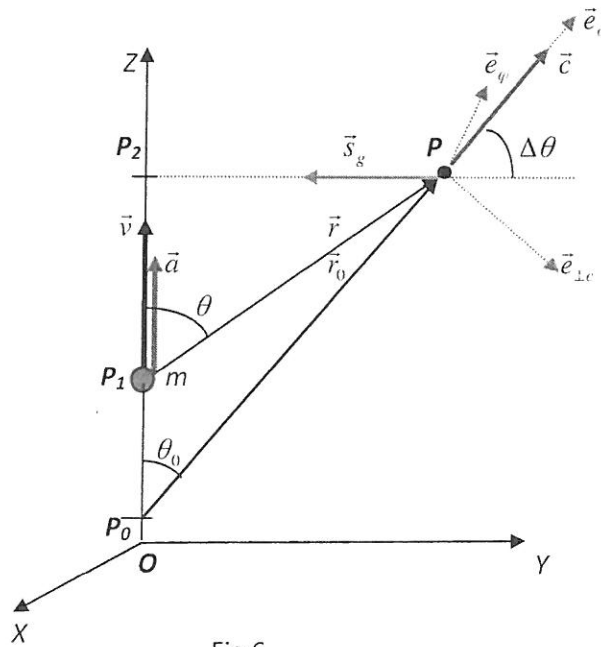


Fig 6

In fig 6 we consider a point mass m that, during a finite time interval, moves with constant acceleration $\vec{a} = a \cdot \vec{e}_z$ relative to the inertial reference frame $OXYZ$. At the moment $t = 0$, m starts - from rest - in the

origin O , and at $t = t$ it passes at the point P_1 . Its velocity is there defined by $\vec{v} = v.\vec{e}_z = a.t.\vec{e}_z$, and its position by $z = \frac{1}{2}.a.t^2 = \frac{1}{2}.v.t$. We suppose that the speed v remains much smaller than the speed of light: $\frac{v}{c} \ll 1$.

The informations that during the infinitesimal time interval $(t, t+dt)$ pass near the fixed point P (whose position relative to the moving mass m is defined by the time dependant position vector $\vec{r}$) have been emitted at the moment $t_0 = t - \Delta t$, when m - with velocity $\vec{v}_0 = v_0.\vec{e}_z = v(t - \Delta t).\vec{e}_z$ - passed at P_0 (the position of P relative to P_0 is defined by the time dependant position vector $\vec{r}_0 = \vec{r}(t - \Delta t)$). Δt , the time interval during which m moves from P_0 to P_1 is the time that the informations need to move - with the speed of light - from P_0 to P . We can conclude that $\Delta t = \frac{r_0}{c}$, and that

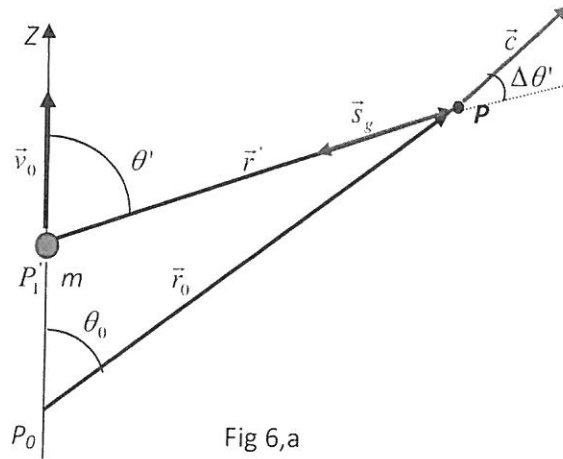
$$v_0 = v(t - \Delta t) = v(t - \frac{r_0}{c}) = v - a.\frac{r_0}{c}.$$

Between the moments $t = t_0$ and $t = t_0 + \Delta t$, m moves from P_0 to P_1 . That movement can be considered as the resultant of

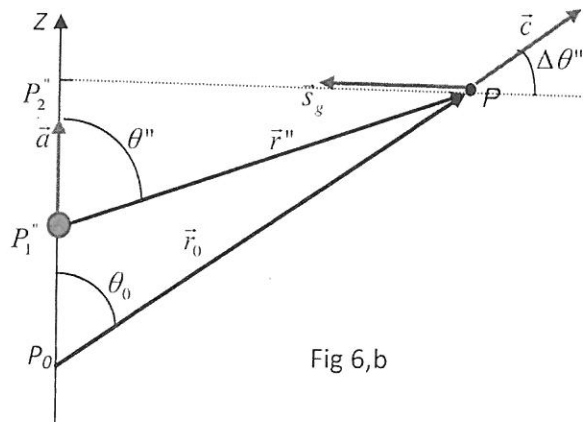
1. a uniform movement with constant speed $v_0 = v(t - \Delta t)$ and
2. a uniformly accelerated movement with constant acceleration a .

1. In fig. 6,a, we consider the case of the point mass m moving with constant speed v_0 along the Z -axis. At the moment $t_0 = t - \Delta t$ m passes at P_0 and at the moment t at P_1 : $P_0P_1 = v_0.\Delta t$. The informations that, during the infinitesimal time interval $(t, t + dt)$, pass near the point P - whose position relative to the uniformly moving mass m at the moment t_0 is defined by the position vector $\vec{r}'$ - have been emitted at the moment t_0 when m passed at P_0 . Their velocity vector $\vec{c}$ is on the line P_0P , their g-index $\vec{s}_g$ points to P_1 :

$$P_0 P_1' = v_0 \cdot \Delta t = v_0 \frac{r_0}{c}$$



2. In fig 6,b we consider the case of the point mass m starting at rest at P_0 and moving with constant acceleration a along the Z-axis.



At the moment $t_0 = t - \Delta t$ it is at P_0 and at the moment t at P_1'' :

$$P_0 P_1'' = \frac{1}{2} \cdot a \cdot (\Delta t)^2$$

The informatons that during the infinitesimal time interval $(t, t + dt)$ pass near the point P (whose position relative to the uniformly accelerated mass m is at t defined by the position vector $\vec{r}''$) have been emitted at t_0 when m was at P_0 . Their velocity vector $\vec{c}$ points to P_0 , their g-index $\vec{s}_g$ to P_2'' . To determine the position of P_2'' , we consider the trajectory of the informatons that at t_0 are emitted in the direction of P , relative to the accelerated reference frame $OX'Y'Z'$ that is anchored to m . (fig 6,c;

$$\alpha = \frac{\pi}{2} - \theta_0)$$

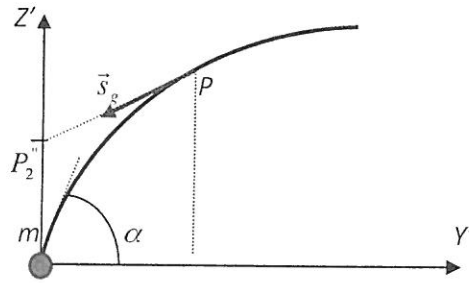


Fig. 6,c

Relative to $OX'Y'Z'$ these informatons are accelerated with an amount $-\vec{a}$: they follow a parabolic trajectory defined by the equation:

$$z' = tg \alpha \cdot y' - \frac{1}{2} \cdot \frac{a}{c^2 \cdot \cos^2 \alpha} \cdot y'^2$$

At the moment $t = t_0 + \Delta t$, when they pass at P , the tangent line to that trajectory cuts the Z' -axis in the point P_2'' , that is defined by:

$$z'_{P_2''} = \frac{1}{2} \cdot a \cdot (\Delta t)^2 = \frac{1}{2} \cdot a \cdot \frac{r_0^2}{c^2}$$

That means that the g-indices of the informations that at the moment t pass at P , point to a point P_2'' on the Z-axis that has a lead of

$$P_1''P_2'' = \frac{1}{2} \cdot a \cdot (\Delta t)^2 = \frac{1}{2} \cdot a \cdot \frac{r_0^2}{c^2}$$

on P_1'' , the actual position of the mass m . And since $P_0P_2'' = P_0P_1'' + P_1''P_2''$, we conclude that:

$$P_0P_2'' = a \cdot \frac{r_0^2}{c^2}$$

In the inertial reference frame $OXYZ$ (fig 6), $\vec{s}_g$ points to the point P_2 on the Z-axis determined by the superposition of the effect of the velocity (1) and the effect of the acceleration (2):

$$P_0P_2 = P_0P_1' + P_0P_2'' = \frac{v_0}{c} \cdot r_0 + \frac{a}{c^2} \cdot r_0^2$$

The carrier line of the g-index $\vec{s}_g$ of an informaton that - relative to the inertial frame $OXYZ$ - at the moment t passes near P forms a "characteristic angle" $\Delta\theta$ with the carrier line of its velocity vector $\vec{c}$, that can be deduced by application of the sine-rule in triangle P_0P_2P (fig 6):

$$\frac{\sin(\Delta\theta)}{P_0P_2} = \frac{\sin(\theta_0 + \Delta\theta)}{r_0}$$

We conclude:

$$\sin(\Delta\theta) = \frac{v_0}{c} \cdot \sin(\theta_0 + \Delta\theta) + \frac{a}{c^2} \cdot r_0 \cdot \sin(\theta_0 + \Delta\theta)$$

From the fact that P_0P_1 - the distance travelled by m during the time interval Δt - can be neglected relative to P_0P - the distance travelled by light in the same interval - it follows that $\theta_0 \approx \theta_0 + \Delta\theta \approx \theta$ and that $r_0 \approx r$

So:

$$\sin(\Delta\theta) \approx \frac{v_0}{c} \cdot \sin\theta + \frac{a}{c^2} \cdot r \cdot \sin\theta$$

We can conclude that the g-index $\vec{s}_g$ of an informaton that at the moment t passes near P , has a longitudinal component, this is a component in the direction of $\vec{c}$ (its velocity vector) and a transversal component, this is a component perpendicular to that direction. It is evident that:

$$\begin{aligned}\vec{s}_g &= -s_g \cdot \cos(\Delta\theta) \cdot \vec{e}_c - s_g \cdot \sin(\Delta\theta) \cdot \vec{e}_{\perp c} \\ &\approx -s_g \cdot \vec{e}_c - s_g \cdot \left(\frac{v_0}{c} \cdot \sin\theta + \frac{a}{c^2} \cdot r \cdot \sin\theta \right) \cdot \vec{e}_{\perp c}\end{aligned}$$

3.8.2 The gravitational field of an accelerated point mass

The informatons that, at the moment t , are passing near the fixed point P - defined by the time dependent position vector $\vec{r}$ - are emitted when m was at P_0 (fig 6). Their velocity $\vec{c}$ is on the same carrier line as $\vec{r}_0 = \overrightarrow{P_0P}$. Their g-spin vector is on the carrier line P_2P . According to §3.8.1, the characteristic angle $\Delta\theta$ - this is the angle between the carrier lines of $\vec{s}_g$ and $\vec{c}$ - has two components:

- a component $\Delta\theta'$ related to the velocity of m at the moment $(t - \frac{r_0}{c})$ when the considered informatons were emitted. In the framework of our assumptions, this component is determined by:

$$\sin(\Delta\theta') = \frac{v(t - \frac{r}{c})}{c} \cdot \sin\theta$$

- a component $\Delta\theta''$ related to the acceleration of m at the moment when they were emitted. This component is, in the framework of our assumptions, determined by:

$$\sin(\Delta\theta'') = \frac{a(t - \frac{r}{c}) \cdot r}{c^2} \cdot \sin\theta$$

The macroscopic effect of the emission of g-information by the accelerated mass m is a gravitational field $(\vec{E}_g, \vec{B}_g)$. We introduce the reference system $(\vec{e}_c, \vec{e}_{\perp c}, \vec{e}_\psi)$ (fig 6).

1. $\vec{E}_g$, the field in P , is defined as the density of the flow of g-information at that point. That density is the rate at which g-information per unit area crosses the elementary surface perpendicular to the direction of movement of the informatons. So $\vec{E}_g$ is the product of N , the density of the flow of informatons at P , with $\vec{s}_g$ their g-index:

$$\vec{E}_g = N \cdot \vec{s}_g$$

According to the postulate of the emission of informatons, the magnitude of $\vec{s}_g$ is the elementary g-information quantity:

$$s_g = \frac{1}{K \cdot \eta_0} = 6,18 \cdot 10^{-60} m^3 s^{-1}$$

and the density of the flow of informatons in P is:

$$N = \frac{\dot{N}}{4\pi \cdot r_0^2} \approx \frac{\dot{N}}{4\pi \cdot r^2} = \frac{K \cdot m}{4\pi \cdot r^2}.$$

Taking this into account that $\frac{1}{\eta_0 \cdot c^2} = v_0$, we obtain:

$$\vec{E}_g = -\frac{m}{4\pi \cdot \eta_0 \cdot r^2} \cdot \vec{e}_c - \left\{ \frac{m}{4\pi \cdot \eta_0 \cdot c \cdot r^2} \cdot v \left(t - \frac{r}{c} \right) \cdot \sin \theta + \frac{v_0 \cdot m}{4\pi \cdot r} \cdot a \left(t - \frac{r}{c} \right) \cdot \sin \theta \right\} \cdot \vec{e}_{\perp c}$$

2. $\vec{B}_g$, the gravitational induction at P , is defined as the density of the cloud of β -information at that point. That density is the product of n , the density of the cloud of informations at P (number per unit volume) with $\vec{s}_\beta$, their β -index:

$$\vec{B}_g = n \cdot \vec{s}_\beta$$

The β -index of an informaton characterizes the information it carries about the state of motion of its emitter; it is defined as:

$$\vec{s}_\beta = \frac{\vec{c} \times \vec{s}_g}{c}$$

And the density of the cloud of informatons in P is related to N , the density of the flow of informatons in that point by:

$$n = \frac{N}{c}$$

So:

$$\vec{B}_g = n \cdot \vec{s}_\beta = \frac{N}{c} \cdot \frac{\vec{c} \times \vec{s}_g}{c} = \frac{\vec{c} \times (N \cdot \vec{s}_g)}{c^2} = \frac{\vec{c} \times \vec{E}_g}{c^2}$$

And with:

$$\vec{E}_g = -\frac{m}{4\pi\eta_0 r^2} \cdot \vec{e}_c - \left\{ \frac{m}{4\pi\eta_0 c r^2} \cdot v \left(t - \frac{r}{c} \right) \cdot \sin \theta + \frac{v_0 m}{4\pi r} \cdot a \left(t - \frac{r}{c} \right) \cdot \sin \theta \right\} \cdot \vec{e}_{\perp c},$$

we finally obtain:

$$\vec{B}_g = -\left\{ \frac{v_0 m}{4\pi r^2} \cdot v \left(t - \frac{r}{c} \right) \cdot \sin \theta + \frac{v_0 m}{4\pi c r} \cdot a \left(t - \frac{r}{c} \right) \cdot \sin \theta \right\} \cdot \vec{e}_\varphi$$

4 THE LAWS OF THE GRAVITATIONAL FIELD - THE LAWS OF G.E.M.

In the space linked to an inertial reference frame O , the gravitational field is characterised by two time dependent vectors: the (effective) g-field $\vec{E}_g$ and the (effective) g-induction $\vec{B}_g$. At an arbitrary point P , these vectors are the results of the superposition of the contributions of the various sources of informatons (the masses) to respectively the density of the flow of g-information and to the cloud of β -information at P :

$$\vec{E}_g = \sum N \cdot \vec{s}_g \quad \text{and} \quad \vec{B}_g = \sum n \cdot \vec{s}_\beta$$

The informatons that - at the moment t - pass with velocity $\vec{c}$ near P contribute with an amount $(N \cdot \vec{s}_g)$ to the instantaneous value of the g-field and with an amount $(n \cdot \vec{s}_\beta)$ to the instantaneous value of the g-induction in that point.

- $\vec{s}_g$ and $\vec{s}_\beta$ respectively are their g-index and their β -index. These are

linked by the relationship: $\vec{s}_\beta = \frac{\vec{c} \times \vec{s}_g}{c}$

- N is the instantaneous value of the density of the flow of informatons with velocity $\vec{c}$ at P and n is the instantaneous value of the density of the cloud of those informatons at that point. N and n are linked by the

relationship: $n = \frac{N}{c}$

4.1 Relations between $\vec{E}_g$ and $\vec{B}_g$ at a matter free point of a gravitational field

At each point where no matter is located - where $\rho_G(x, y, z; t) = \vec{J}_G(x, y, z; t) = 0$ - the following statements are valid.

4.1.1 At a matter free point P of a gravitational field, the spatial variation of $\vec{E}_g$ obeys the law: $\text{div} \vec{E}_g = 0$

This statement is the expression of the law of conservation of g-information.

The fact that the rate at which g-information flows inside a closed empty space must be equal to the rate at which it flows out, can be expressed as:

$$\oiint_S \vec{E}_g \cdot \vec{dS} = 0$$

So (theorem of Ostrogradsky) [5]:

$$\text{div} \vec{E}_g = 0$$

4.1.2 At a matter free point P of a gravitational field, the spatial variation of $\vec{B}_g$ obeys the law: $\text{div} \vec{B}_g = 0$

This statement is the expression of the fact that the β -index of an informaton is always perpendicular to its g -index $\vec{s}_g$ and to its velocity $\vec{c}$.

In fig 7, we consider the flow of informatons that - at the moment t - pass with velocity $\vec{c}$ near the point P . An informaton that at the moment t passes at P is at the moment $(t + dt)$ at Q : $PQ = c \cdot dt$. At P , the instantaneous value of the density of the considered flow of informatons is represented by N , the instantaneous value of the density of the cloud that they constitute by n , and the instantaneous value of their characteristic angle by $\Delta\theta$. In the coordinate system $PXYZ$ $\vec{s}_g$ and $\vec{s}_\beta$ can be expressed as:

$$\vec{s}_g = -s_g \cdot \vec{e}_x$$

and

$$\vec{s}_\beta = \frac{\vec{c} \times \vec{s}_g}{c} = s_g \cdot \sin(\Delta\theta) \cdot \vec{e}_z$$

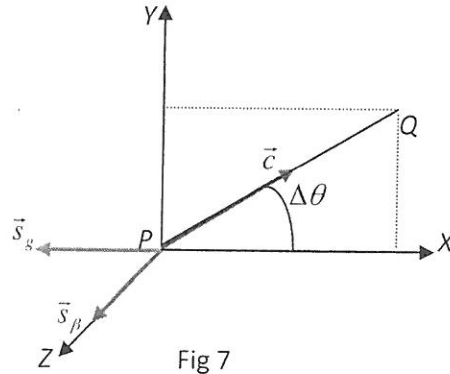


Fig 7

The contribution of the considered informatons to the g-field at P is $\vec{E}_g = N \cdot \vec{s}_g$ and to the g-induction it is $\vec{B}_g = n \cdot \vec{s}_\beta$

From mathematics [5] we know:

$$\text{div} \vec{B}_g = \text{div}(n \cdot \vec{s}_\beta) = \text{grad}(n) \cdot \vec{s}_\beta + n \cdot \text{div}(\vec{s}_\beta)$$

- $\text{grad}(n) \cdot \vec{s}_\beta = 0$ because $\text{grad}(n)$ is perpendicular to $\vec{s}_\beta$. Indeed n changes only in the direction of the flow of informatons, so $\text{grad}(n)$ has the same orientation as $\vec{c}$:

$$\text{grad}(n) = \frac{n_Q - n_P}{PQ} \cdot \frac{\vec{c}}{c}$$

- $n \cdot \text{div}(\vec{s}_\beta) = 0$. According to the definition: $\text{div}(\vec{s}_\beta) = \frac{\oiint \vec{s}_\beta \cdot \vec{dS}}{dV}$ (dV is the enclosed infinitesimal volume). Calculating the double integral over the closed surface S formed by the infinitesimal surfaces $dS = dz \cdot dy$ which are at P and at Q perpendicular to the X -axis and by the tube that connects the edges of these surfaces, it is obvious that:

$$\text{div}(\vec{s}_\beta) = \frac{\oiint \vec{s}_\beta \cdot \vec{dS}}{dV} = 0$$

Both terms of the expression of $\text{div} \vec{B}_g$ are zero, so $\text{div} \vec{B}_g = 0$, what implies (theorem of Ostrogradsky) that for every closed surface S in a gravitational field:

$$\oiint_S \vec{B}_g \cdot \vec{dS} = 0$$

4.1.3 At a matter free point P of a gravitational field, the spatial variation of $\vec{E}_g$ and the rate at which $\vec{B}_g$ is changing are connected by the relation:

$$\text{rot} \vec{E}_g = - \frac{\partial \vec{B}_g}{\partial t}$$

This statement is the expression of the fact that any change of the product $n \cdot \vec{s}_\beta$ in a point of a gravitational field is related to a variation of the product $N \cdot \vec{s}_g$ in the vicinity of that point.

We consider again $\vec{E}_g$ and $\vec{B}_g$, the contributions to the g-field and to the g-induction at the point P of the informatons which pass - at the moment t - with velocity $\vec{c}$ near that point (fig 7).

$$\vec{E}_g = N \cdot \vec{s}_g = -N \cdot s_g \cdot \vec{e}_x \quad \text{and} \quad \vec{B}_g = n \cdot \vec{s}_\beta = n \cdot \frac{\vec{c} \times \vec{s}_g}{c} = n \cdot s_g \cdot \sin(\Delta\theta) \cdot \vec{e}_z$$

We investigate the relationship between

$$\text{rot} \vec{E}_g = \{ \text{grad}(N) \times \vec{s}_g \} + N \cdot \text{rot}(\vec{s}_g) \quad \text{and} \quad \frac{\partial \vec{B}_g}{\partial t} = \frac{\partial n}{\partial t} \cdot \vec{s}_\beta + n \cdot \frac{\partial \vec{s}_\beta}{\partial t}$$

- $\{ \text{grad}(N) \times \vec{s}_g \}$, the first term of $\text{rot} \vec{E}_g$, describes the component of $\text{rot} \vec{E}_g$ caused by the spatial variation of N in the vicinity of P when $\Delta\theta$ remains constant.

N has the same value in all points of the infinitesimal surface that, at P , is perpendicular to the flow of informatons. So $\text{grad}(N)$ is parallel to $\vec{c}$ and its magnitude is the increase of the magnitude of N per unit length. With $N_P = N$, $N_Q = N + dN$ and $PQ = c \cdot dt$, $\text{grad}(N)$ is determined by:

$$\text{grad}(N) = \frac{N_Q - N_P}{PQ} \frac{\vec{c}}{c} = \frac{dN}{c \cdot dt} \cdot \frac{\vec{c}}{c}$$

And:

$$\text{grad}(N) \times \vec{s}_g = \frac{dN}{c \cdot dt} \cdot \frac{\vec{c}}{c} \times \vec{s}_g = \frac{dN}{c \cdot dt} \cdot \vec{s}_\beta$$

From the fact that the density of the flow of informatons at Q at the moment t is equal to the density of that flow at P at $(t - dt)$, it follows:

If $N_P(t) = N$, then $N_P(t - dt) = N_Q(t) = N + dN$

The rate at which N_P changes at the moment t is:

$$\frac{\partial N}{\partial t} = \frac{N_P(t) - N_P(t - dt)}{dt} = -\frac{dN}{dt}$$

And since: $\frac{N}{c} = n$: $\frac{1}{c} \frac{dN}{dt} = -\frac{1}{c} \frac{\partial N}{\partial t} = -\frac{\partial n}{\partial t}$

We can conclude (1):

$$\text{grad}(N) \times \vec{s}_g = -\frac{\partial n}{\partial t} \vec{s}_\beta$$

- $\{N \cdot \text{rot}(\vec{s}_g)\}$, the second term of $\text{rot} \vec{E}_g$, describes the component of $\text{rot} \vec{E}_g$ caused by the spatial variation of $\Delta\theta$ - the orientation of the g-index in the vicinity of P - when N remains constant. At the moment t , $(\Delta\theta)_P$ - the characteristic angle of the informatons that pass near P - differs from $(\Delta\theta)_Q$ - the characteristic angle of the informatons that pass near Q . If $(\Delta\theta)_P = \Delta\theta$, then $(\Delta\theta)_Q = \Delta\theta + d(\Delta\theta)$ (fig 8).

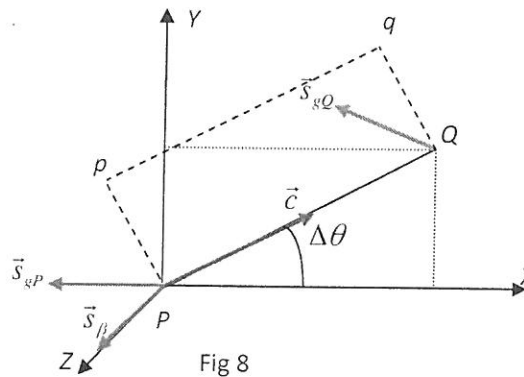


Fig 8

For the calculation of $rot(\vec{s}_g) = \frac{\oint \vec{s}_g \cdot d\vec{l}}{dS}$ (dS is the encircled area), we calculate $\oint \vec{s}_g \cdot d\vec{l}$ along the closed path $PQqpP$ that encircles $dS = PQ \cdot Pp = c \cdot dt \cdot Pp$. (PQ and qp are parallel to the flow of the informatons, Qq and pP are perpendicular to it).

$$N \cdot rot(\vec{s}_g) = N \cdot \frac{s_g \cdot \sin\{\Delta\theta + d(\Delta\theta)\} \cdot Qq - s_g \cdot \sin(\Delta\theta) \cdot Pp}{c \cdot dt \cdot Pp} \cdot \vec{e}_z$$

From the fact that the characteristic angle of the informatons at Q at the moment t is equal to the characteristic angle of the informatons at P at the moment $(t - dt)$, it follows:

$$\text{If } (\Delta\theta)_P(t) = \Delta\theta, \text{ then } (\Delta\theta)_P(t - dt) = (\Delta\theta)_Q(t) = \Delta\theta + d(\Delta\theta)$$

The rate at which $\sin(\Delta\theta)$ in P changes at the moment t , is:

$$\frac{\partial\{\sin(\Delta\theta)\}}{\partial t} = \frac{\sin(\Delta\theta) - \sin\{\Delta\theta + d(\Delta\theta)\}}{dt} = -\frac{d\{\sin(\Delta\theta)\}}{dt}$$

And since $N = c \cdot n$, we obtain:

$$N \cdot rot(\vec{s}_g) = N \cdot s_g \cdot \frac{\sin\{\Delta\theta + d(\Delta\theta)\} - \sin(\Delta\theta)}{c \cdot dt}$$

$$\text{Finally: } N \cdot rot\vec{E}_g = \frac{\partial}{\partial t} \{n \cdot s_g \cdot \sin(\Delta\theta) \cdot \vec{e}_z\} = -n \cdot \frac{\partial \vec{s}_g}{\partial t} \quad (II)$$

Combining the results (I) and (II), we obtain:

$$rot\vec{E}_g = grad(N_g) \times \vec{s}_g + N_g \cdot rot(\vec{s}_g) = -\left(\frac{\partial n_g}{\partial t} \cdot \vec{s}_g + n_g \cdot \frac{\partial \vec{s}_g}{\partial t}\right) = -\frac{\partial \vec{B}_g}{\partial t}$$

The relation $rot \vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$ implies (theorem of Stokes [5]): In a gravitational field, the rate at which the surface integral of $\vec{B}_g$ over a surface S changes is equal and opposite to the line integral of $\vec{E}_g$ over its boundary L :

$$\oint_L \vec{E}_g \cdot d\vec{l} = - \iint_S \frac{\partial \vec{B}_g}{\partial t} \cdot d\vec{S} = - \frac{\partial}{\partial t} \iint_S \vec{B}_g \cdot d\vec{S} = - \frac{\partial \Phi_B}{\partial t}$$

The orientation of the surface vector $d\vec{S}$ is linked to the orientation of the path on L by the "rule of the corkscrew". $\Phi_B = \iint_S \vec{B}_g \cdot d\vec{S}$ is called the " β -information-flux through S ".

4.1.4 At a matter free point P of a gravitational field, the spatial variation of $\vec{B}_g$ and the rate at which $\vec{E}_g$ is changing are connected by the relation:

$$rot \vec{B}_g = \frac{1}{c^2} \frac{\partial \vec{E}_g}{\partial t}$$

This statement is the expression of the fact that any change of the product $N \cdot \vec{s}_g$ in a point of a gravitational field is related to a variation of the product $n \cdot \vec{s}_g$ in the vicinity of that point.

We consider again $\vec{E}_g$ and $\vec{B}_g$, the contributions of the informations that - at the moment t - pass near P with velocity $\vec{c}$, to the g-field and to the g-induction in that point. (fig 8).

$$\vec{E}_g = N \cdot \vec{s}_g = -N \cdot s_g \cdot \vec{e}_x \quad \text{and} \quad \vec{B}_g = n \cdot \vec{s}_\beta = n \cdot \frac{\vec{c} \times \vec{s}_g}{c} = n \cdot s_g \cdot \sin(\Delta\theta) \cdot \vec{e}_z$$

And first we note that $\vec{s}_g = -s_g \cdot \vec{e}_x$ and that $\frac{\partial \vec{s}_g}{\partial t} = s_g \cdot \frac{\partial(\Delta\theta)}{\partial t} \cdot \vec{e}_y$

We investigate the relationship between

$$\text{rot}\vec{B}_g = \{\text{grad}(n) \times \vec{s}_\beta\} + n.\text{rot}(\vec{s}_\beta) \quad \text{and} \quad \frac{\partial \vec{E}_g}{\partial t} = \frac{\partial N}{\partial t}.\vec{s}_g + N.\frac{\partial \vec{s}_g}{\partial t}$$

1°. First we calculate: $\text{rot}\vec{B}_g = \{\text{grad}(n) \times \vec{s}_\beta\} + n.\text{rot}(\vec{s}_\beta)$

- $\{\text{grad}(n) \times \vec{s}_\beta\}$, the first term of $\text{rot}\vec{B}_g$, describes the component of $\text{rot}\vec{B}_g$ caused by the spatial variation of n in the vicinity of P when $\Delta\theta$ remains constant.

n has the same value in all points of the infinitesimal surface that, at P , is perpendicular to the flow of informatons. So $\text{grad}(n)$ is parallel to $\vec{c}$ and its magnitude is the increase of the magnitude of n per unit length.

With $n_P = n$, $n_Q = n + dn$ and $PQ = c.dt$, $\text{grad}(n)$ is determined by:

$$\text{grad}(n) = \frac{n_Q - n_P}{PQ} \frac{\vec{c}}{c} = \frac{dn}{c.dt} \cdot \frac{\vec{c}}{c}$$

The vector $\{\text{grad}(n) \times \vec{s}_\beta\}$ is perpendicular to the plane determined by $\vec{c}$ and $\vec{s}_\beta$. So, it lies in the XY -plane and is there perpendicular to $\vec{c}$. Taking into account the definition of vectorial product, we obtain (fig 8):

$$\text{grad}(n) \times \vec{s}_\beta = -\frac{dn}{c.dt} \cdot s_\beta \cdot \vec{e}_{\perp c} = -\frac{dn}{c.dt} \cdot s_g \cdot \sin(\Delta\theta) \cdot \vec{e}_{\perp c}$$

From the fact that the density of the cloud of informatons at Q at the moment t is equal to the density of that cloud at P at the

moment $(t - dt)$, it follows:

$$\text{If } n_P(t) = n, \text{ then } n_P(t - dt) = n_Q(t) = n + dn$$

The rate at which n_P changes at the moment t is:

$$\frac{\partial n}{\partial t} = \frac{1}{c} \cdot \frac{\partial N}{\partial t} = \frac{n_P(t) - n_P(t - dt)}{dt} = \frac{n_P(t) - n_Q(t)}{dt} = - \frac{dn}{dt}$$

And, taking into account that $n = \frac{N}{c}$, we obtain (1):

$$\text{grad}(n) \times \vec{s}_\beta = \frac{1}{c} \cdot \frac{\partial n}{\partial t} \cdot s_g \cdot \sin(\Delta\theta) \cdot \vec{e}_{\perp c} = \frac{1}{c^2} \cdot \frac{\partial N}{\partial t} \cdot s_g \cdot \sin(\Delta\theta) \cdot \vec{e}_{\perp c}$$

- The term $\{n \cdot \text{rot}(\vec{s}_\beta)\}$ is the component of $\text{rot}\vec{B}_g$ caused by the spatial variation of $\vec{s}_\beta$ in the vicinity of P when n remains constant. The fact that $\vec{s}_{\beta Q} \neq \vec{s}_{\beta P}$ at the moment t , follows from the fact that, at that moment, $(\Delta\theta)_P$ - the characteristic angle of the informatons that pass in P - differs from $(\Delta\theta)_Q$ - the characteristic angle of the informatons that pass in Q . If $(\Delta\theta)_P = \Delta\theta$, than $(\Delta\theta)_Q = \Delta\theta + d(\Delta\theta)$. From the definition of $\text{rot}\vec{F}$ [5], it follows (fig 9):

$$\begin{aligned} \text{rot}(\vec{s}_\beta) &= \frac{\oint \vec{s}_\beta \cdot d\vec{l}}{dS} \cdot \vec{e}_{\perp c} = \frac{s_g \cdot \sin(\Delta\theta) \cdot Pp - s_g \cdot \sin\{(\Delta\theta) + d(\Delta\theta)\} \cdot qQ}{c \cdot dt \cdot Pp} \cdot \vec{e}_{\perp c} \\ &= -s_g \frac{d \sin(\Delta\theta)}{c \cdot dt} \cdot \vec{e}_{\perp c} \end{aligned}$$

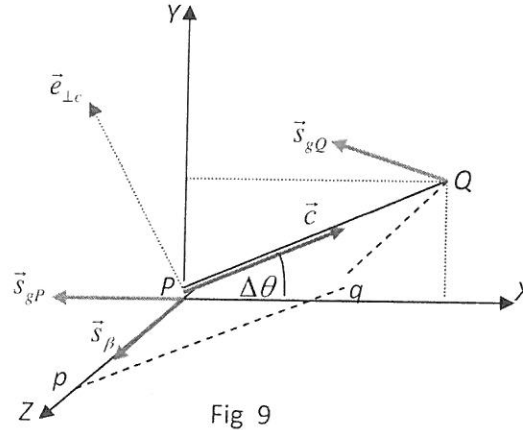


Fig 9

From the fact that the characteristic angle of the informatons at Q at the moment t is equal to the characteristic angle of the informatons at P at the moment $(t - dt)$, it follows that if

$$(\Delta\theta)_P(t) = \Delta\theta, \text{ then } (\Delta\theta)_P(t - dt) = (\Delta\theta)_Q(t) = \Delta\theta + d(\Delta\theta)$$

The rate at which $\sin(\Delta\theta)$ in P changes at the moment t , is:

$$\frac{\partial\{\sin(\Delta\theta)\}}{\partial t} = \frac{\sin(\Delta\theta) - \sin\{\Delta\theta + d(\Delta\theta)\}}{dt} = -\frac{d\{\sin(\Delta\theta)\}}{dt}$$

$$\text{Further : } \frac{\partial}{\partial t}\{\sin(\Delta\theta)\} = \cos(\Delta\theta) \cdot \frac{\partial(\Delta\theta)}{\partial t} \quad \text{and} \quad n = \frac{N}{c}$$

$$\text{Finally, we obtain (II): } n \cdot \text{rot}(\vec{s}_\beta) = \frac{1}{c^2} \cdot N \cdot s_g \cdot \cos(\Delta\theta) \cdot \frac{\partial(\Delta\theta)}{\partial t} \cdot \vec{e}_{\perp c}$$

Combining the results (I) and (II), we obtain:

$$rot \vec{B}_g = \frac{1}{c^2} \cdot \left\{ \frac{\partial N}{\partial t} s_g \cdot \sin(\Delta\theta) + N s_g \cdot \cos(\Delta\theta) \cdot \frac{\partial(\Delta\theta)}{\partial t} \right\} \vec{e}_{\perp c}$$

2°. Next we calculate:

$$\frac{\partial \vec{E}_g}{\partial t} = \frac{\partial N}{\partial t} \vec{s}_g + N \cdot \frac{\partial \vec{s}_g}{\partial t} = -\frac{\partial N}{\partial t} s_g \cdot \vec{e}_x + N s_g \cdot \frac{\partial(\Delta\theta)}{\partial t} \cdot \vec{e}_y$$

Taking into account:

$$\vec{e}_x = \cos(\Delta\theta) \cdot \vec{e}_c - \sin(\Delta\theta) \cdot \vec{e}_{\perp c} \quad \text{and} \quad \vec{e}_y = \sin(\Delta\theta) \cdot \vec{e}_c + \cos(\Delta\theta) \cdot \vec{e}_{\perp c}$$

we obtain:

$$\begin{aligned} \frac{\partial \vec{E}_g}{\partial t} = & \left[-\frac{\partial N}{\partial t} s_g \cdot \cos(\Delta\theta) + N s_g \cdot \frac{\partial(\Delta\theta)}{\partial t} \cdot \sin(\Delta\theta) \right] \vec{e}_c \\ & + \left[\frac{\partial N}{\partial t} s_g \cdot \sin(\Delta\theta) + N s_g \cdot \frac{\partial(\Delta\theta)}{\partial t} \cdot \cos(\Delta\theta) \right] \vec{e}_{\perp c} \end{aligned}$$

From the first law of the gravitational field (§4.1.1), it follows that the component of $\frac{\partial \vec{E}_g}{\partial t}$ in the direction of $\vec{e}_c$ is zero. Indeed.

- We know (§4.1.3): $grad(N) = -\frac{1}{c^2} \cdot \frac{\partial N}{\partial t} \cdot \vec{c}$, so:

$$grad(N) \cdot \vec{s}_g = \frac{1}{c} \cdot \frac{\partial N}{\partial t} s_g \cdot \cos(\Delta\theta) \quad (III)$$

- We determine $div(\vec{s}_g) = \frac{\oiint \vec{s}_g \cdot d\vec{S}}{dV}$ (IV). For that purpose, we calculate the double integral over the closed surface S formed by the infinitesimal surfaces dS which are at P and Q perpendicular to the flow of informatons (perpendicular to $\vec{c}$) and by the tube that

connects the edges of these surfaces (and that is parallel to $\vec{c}$).
 $dV = c \cdot dt \cdot dS$ is the infinitesimal volume enclosed by S :

$$\begin{aligned} \text{div}(\vec{s}_g) &= \frac{\oint \vec{s}_g \cdot d\vec{S}}{dV} = \frac{s_g \cdot dS \cdot \cos(\Delta\theta) - s_g \cdot dS \cdot \cos\{\Delta\theta + d(\Delta\theta)\}}{dS \cdot c \cdot dt} \\ &= -\frac{1}{c} \cdot s_g \cdot \frac{d\{\cos(\Delta\theta)\}}{dt} = -\frac{1}{c} \cdot s_g \cdot \sin(\Delta\theta) \cdot \frac{\partial(\Delta\theta)}{\partial t} \end{aligned}$$

So (IV):

$$N \cdot \text{div}(\vec{s}_g) = -\frac{1}{c} \cdot N \cdot s_g \cdot \sin(\Delta\theta) \cdot \frac{\partial(\Delta\theta)}{\partial t}$$

According to the first law of the gravitational field (V):

$$-\text{div}\vec{E}_g = -\text{div}(N \cdot \vec{s}_g) = -\text{grad}(N) \cdot \vec{s}_g - N \cdot \text{div}(\vec{s}_g) = 0$$

Substitution of (III) and (IV) in (V):

$$-\text{div}\vec{E}_g = -\frac{1}{c} \cdot \frac{\partial N}{\partial t} \cdot s_g \cdot \cos(\Delta\theta) + \frac{1}{c} \cdot N \cdot s_g \cdot \sin(\Delta\theta) \cdot \frac{\partial(\Delta\theta)}{\partial t} = 0$$

So, the component of $\frac{\partial \vec{E}_g}{\partial t}$ in the direction of $\vec{e}_c$ is zero, and:

$$\frac{\partial \vec{E}_g}{\partial t} = \left\{ \frac{\partial N}{\partial t} \cdot s_g \cdot \sin(\Delta\theta) + N \cdot s_g \cdot \frac{\partial(\Delta\theta)}{\partial t} \cdot \cos(\Delta\theta) \right\} \cdot \vec{e}_{\perp c}$$

3°. Conclusion: From 1° en 2° follows: $\text{rot}\vec{B}_g = \frac{1}{c^2} \frac{\partial \vec{E}_g}{\partial t}$

This relation implies (theorem of Stokes): *In a gravitational field, the rate at which the surface integral of $\vec{E}_g$ over a surface S changes is proportional to the line integral of $\vec{B}_g$ over its boundary L :*

$$\oint \vec{B}_g \cdot d\vec{l} = \frac{1}{c^2} \iint_S \frac{\partial \vec{E}_g}{\partial t} \cdot d\vec{S} = \frac{1}{c^2} \frac{\partial}{\partial t} \iint_S \vec{E}_g \cdot d\vec{S} = \frac{1}{c^2} \frac{\partial \Phi_G}{\partial t}$$

The orientation of the surface vector $d\vec{S}$ is linked to the orientation of the path on L by the "rule of the corkscrew". $\Phi_G = \iint_S \vec{E}_g \cdot d\vec{S}$ is called the "g-information-flux through S ".

4.2 Relations between $\vec{E}_g$ and $\vec{B}_g$ in a point of a gravitational field

The volume-element at a point P inside a mass continuum is in any case an emitter of g-information and, if the mass is moving, also a source of β -information. According to §2.3, the instantaneous value of ρ_G - the mass density at P - contributes to the instantaneous value of $\text{div} \vec{E}_g$ in that point with an amount $-\frac{\rho_G}{\eta_0}$; and according to §3.7 the instantaneous value of $\vec{J}_G$ - the mass flow density - contributes to the instantaneous value of $\text{rot} \vec{B}_g$ at P with an amount $-\nu_0 \vec{J}_G$ (§3.7).

Generally, at a point of a gravitational field - linked to an inertial reference frame O - one must take into account the contributions of the local values of $\rho_G(x, y, z; t)$ and of $\vec{J}_G(x, y, z; t)$. This results in the generalization and expansion of the laws in a mass free point. By superposition we obtain:

4.2.1 At a point P of a gravitational field, the spatial variation of $\vec{E}_g$

obeys the law: $\text{div} \vec{E}_g = -\frac{\rho_G}{\eta_0}$

In integral form: $\Phi_G = \oint_S \vec{E}_g \cdot \vec{dS} = -\frac{1}{\eta_0} \cdot \iiint_G \rho_G dV$

4.2.2 At a point P of a gravitational field, the spatial variation of $\vec{B}_g$ obeys the law: $\text{div} \vec{B}_g = 0$

In integral form: $\Phi_B = \oint_S \vec{B}_g \cdot \vec{dS} = 0$

4.2.3 At a point P of a gravitational field, the spatial variation of $\vec{E}_g$ and the rate at which $\vec{B}_g$ is changing are connected by the relation:

$$\text{rot} \vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$$

In integral form: $\oint \vec{E}_g \cdot \vec{dl} = -\iint_S \frac{\partial \vec{B}_g}{\partial t} \cdot \vec{dS} = -\frac{\partial}{\partial t} \iint_S \vec{B}_g \cdot \vec{dS} = -\frac{\partial \Phi_B}{\partial t}$

4.2.4 At a point P of a gravitational field, the spatial variation of $\vec{B}_g$ and the rate at which $\vec{E}_g$ is changing are connected by the relation:

$$\text{rot} \vec{B}_g = \frac{1}{c^2} \frac{\partial \vec{E}_g}{\partial t} - \nu_0 \cdot \vec{J}_G$$

In integral form:

$$\oint \vec{B}_g \cdot \vec{dl} = \frac{1}{c^2} \iint_S \frac{\partial \vec{E}_g}{\partial t} \cdot \vec{dS} - \nu_0 \cdot \iint_S \vec{J}_G \cdot \vec{dS} = \frac{1}{c^2} \cdot \frac{\partial}{\partial t} \iint_S \vec{E}_g \cdot \vec{dS} - \nu_0 \cdot \iint_S \vec{J}_G \cdot \vec{dS}$$

These are the laws of Heaviside-Maxwell or the laws of GEM

5 THE INTERACTION BETWEEN MASSES

5.1 The interaction between masses at rest

We consider a set of point masses anchored in an inertial reference frame $\mathcal{O}$. They create and maintain a gravitational field that at each point of the space linked to $\mathcal{O}$ is completely determined by the vector $\vec{E}_g$. Each mass is “immersed” in a cloud of g-information. At every point, except its own anchorage, each mass contributes to the construction of that cloud.

Let us consider the mass m anchored at P . If the other masses were not there, then m would be at the centre of a perfectly spherical cloud of g-information. In reality this is not the case: the emission of g-information by the other masses is responsible for the disturbance of that “characteristic symmetry”. Because $\vec{E}_g$ at P represents the intensity of the flow of g-information sent to P by the other masses, the extent of disturbance of that characteristic symmetry in the direct vicinity of m is determined by $\vec{E}_g$ at P .

If it was free to move, the point mass m could restore the characteristic symmetry of the g-information cloud in his direct vicinity: it would suffice to accelerate with an amount $\vec{a} = \vec{E}_g$. Accelerating in this way has the effect that the extern field disappears in the origin of the reference frame anchored to m . If it accelerates that way, the mass becomes “blind” for the g-information sent to P by the other masses, it “sees” only its own spherical g-information cloud.

These insights are expressed in the following postulate.

5.1.1 The postulate of the gravitational action

A free point mass m at a point of a gravitational field acquires an acceleration $\vec{a} = \vec{E}_g$ so that the characteristic symmetry of the g-information cloud in its direct vicinity is conserved. A point mass that is anchored in a gravitational field cannot accelerate. In that case it *tends* to move. We can conclude that:

A point mass anchored at a point of a gravitational field is subjected to a tendency to move in the direction defined by $\vec{E}_g$, the g-field in that point. Once the anchorage is broken, the mass acquires a vectorial acceleration $\vec{a}$ that equals $\vec{E}_g$.

5.1.2 The gravitational force - the concept force

A point mass m , anchored at a point P of a gravitational field, experiences an action because of that field, an action that is compensated by the anchorage.

- That action is proportional to the extent to which the characteristic symmetry of the own gravitational field of m in the vicinity of P is disturbed by the extern g-field, thus to the value of $\vec{E}_g$ at P .
- It depends also on the magnitude of m . Indeed, the g-information cloud created and maintained by m is more compact if m is greater. That implies that the disturbing effect on the spherical symmetry around m by the extern g-field $\vec{E}_g$ is smaller when m is greater. Thus, to impose the acceleration $\vec{a} = \vec{E}_g$, the action of the gravitational field on m must be greater when m is greater.

We can conclude that the action that tends to accelerate a point mass m in a gravitational field must be proportional to $\vec{E}_g$, the g-field to which the mass is exposed; and to m , the magnitude of the mass. We represent that action by $\vec{F}_g$ and we call this vectorial quantity "the force developed by the g-field on the mass" or the *gravitational force* on m . We define it by the relation:

$$\vec{F}_g = m.\vec{E}_g$$

A mass anchored at a point P cannot accelerate, what implies that the effect of the anchorage must compensate the gravitational force. This means that the disturbance of the characteristic symmetry around P by $\vec{E}_g$ must be cancelled by the g-information flow created and maintained by the anchorage. The density of that flow at P must be equal and

opposite to $\vec{E}_g$. It cannot be otherwise than that the anchorage exerts an action on m that is exactly equal and opposite to the gravitational force. That action is called a *reaction force*. This discussion leads to the following insight:

Each phenomenon that disturbs the characteristic symmetry of the cloud of g-information around a point mass, exerts a force on that mass.

Between the gravitational force on a mass m and the local field strength exists the following relationship:

$$\vec{E}_g = \frac{\vec{F}_g}{m}$$

So, the acceleration imposed to the mass by the gravitational force is:

$$\vec{a} = \frac{\vec{F}_g}{m}$$

Considering that the gravitational force is nothing but a special force, we can conclude that this relation can be generalized.

The relation between a force $\vec{F}$ and the acceleration $\vec{a}$ that it imposes to a free mass m is:

$$\vec{F} = m \cdot \vec{a}$$

5.1.3 Newtons universal law of gravitation

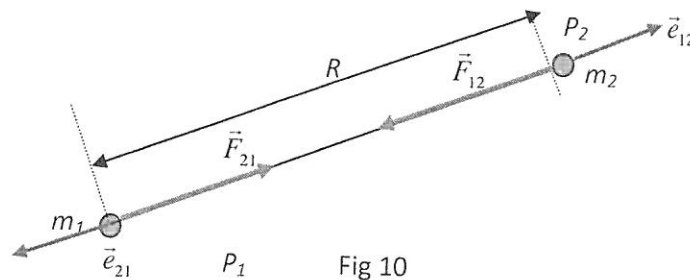


Fig 10

In fig 10 we consider two point masses m_1 and m_2 anchored at the points P_1 and P_2 of an inertial frame. m_1 creates and maintains a gravitational field that at P_2 is defined by the g-field: $\vec{E}_{g2} = -\frac{m_1}{4\pi\eta_0} \cdot \vec{e}_{12}$

This field exerts a gravitational force on m_2 :

$$\vec{F}_{12} = m_2 \cdot \vec{E}_{g2} = -\frac{m_1 \cdot m_2}{4\pi\eta_0} \cdot \vec{e}_{12}$$

In a similar manner we find $\vec{F}_{21}$: $\vec{F}_{21} = -\frac{m_1 \cdot m_2}{4\pi\eta_0} \cdot \vec{e}_{21} = -\vec{F}_{12}$

This is the mathematical formulation of Newtons universal law of gravitation.

5.2. The interaction between moving masses

We consider a number of point masses moving relative to an inertial reference frame O . They create and maintain a gravitational field that at each point of the space linked to O is defined by the vectors $\vec{E}_g$ and $\vec{B}_g$. Each mass is "immersed" in a cloud of informatons carrying both g- and β -information. In each point, except its own position, each mass contributes to the construction of that cloud.

Let us consider the mass m that, at the moment t , goes through the point P with velocity $\vec{v}$.

- If the other masses were not there, according to §3.2 $\vec{E}_g$ - the g-field near m (the "own" g-field of m) - would be symmetric relative to the carrier line of the vector $\vec{v}$. Indeed, the g-indices of the informatons emitted by m during the time interval $(t - \Delta t, t + \Delta t)$ are all directed to that line. In reality that symmetry is disturbed by the g-information that the other masses send to P . $\vec{E}_g$, the instantaneous value of the g-field at P , defines the extent to which this occurs.

- If the other masses were not there, according to §3.4 $\vec{B}_g'$ - the β -field near m (the "own" β -field of m) - would "rotate" around the carrier line of the vector $\vec{v}$. The vectors defining the pseudo-g-field $\vec{E}_g'' = \vec{v} \times \vec{B}_g'$ defined by the vector product of $\vec{v}$ with the g-induction that characterizes the "own" β -field of m , would - just like $\vec{E}_g'$ - be symmetric relative to the carrier line of the vector $\vec{v}$. In reality this symmetry is disturbed by the β -information sent to P by the other masses. The vector product $(\vec{v} \times \vec{B}_g)$ of the instantaneous values of the velocity of m and of the g-induction at P , characterizes the extent to which this occurs.

So, the *characteristic symmetry* of the cloud of information around a moving mass (the "own" gravitational field) is disturbed by $\vec{E}_g$ regarding the "own" g-field; and by $(\vec{v} \times \vec{B}_g)$ regarding the "own" β -field. If it was free to move, the point mass m could restore the characteristic symmetry in its direct vicinity by accelerating relative to its proper inertial reference frame O' with an amount $\vec{a}' = \vec{E}_g + (\vec{v} \times \vec{B}_g)$. In that manner it would become "blind" for the disturbance of symmetry of the gravitational field in its direct vicinity. These insights form the basis of the following postulate.

5.2.1 The postulate of the gravitational action

A point mass m , moving with velocity $\vec{v}$ in a gravitational field $(\vec{E}_g, \vec{B}_g)$, tends to become blind for the influence of that field on the symmetry of its "own" g-field. If it is free to move, it will accelerate relative to its proper inertial reference frame with an amount $\vec{a}'$:

$$\vec{a}' = \vec{E}_g + (\vec{v} \times \vec{B}_g)$$

* The proper inertial reference frame O' of the point mass m is the reference frame that at each moment t moves relative to O with the same velocity as m .

5.2.2 The gravitational force

The action of the gravitational field $(\vec{E}_g, \vec{B}_g)$ on a point mass that is moving with velocity $\vec{v}$ relative to the inertial reference frame $\mathcal{O}$, is called the *gravitational force* $\vec{F}_G$ on that mass. In extension of §5.1.2 we define $\vec{F}_G$ as:

$$\vec{F}_G = m_0 [\vec{E}_g + (\vec{v} \times \vec{B}_g)]$$

m_0 is the rest mass of the point mass: it is the mass that determines the rate at which it emits informations in the space linked to $\mathcal{O}$. If it is free to move, the effect of $\vec{F}_G$ on the point mass m is that it will be accelerated relative to the proper inertial reference frame $\mathcal{O}'$ with an amount $\vec{a}'$. This acceleration can be decomposed in a tangential ($\vec{a}'_T$) and a normal component ($\vec{a}'_N$):

$$\vec{a}'_T = a'_T \vec{e}_T \quad \text{and} \quad \vec{a}'_N = a'_N \vec{e}_N$$

where $\vec{e}_T$ and $\vec{e}_N$ are the unit vectors, respectively along the tangent and along the normal to the path of the point mass in $\mathcal{O}'$ (and in $\mathcal{O}$).

We express a'_T en a'_N in function of the characteristics of the motion in the reference system $\mathcal{O}$ [6]:

$$a'_T = \frac{1}{(1 - \beta^2)^{\frac{3}{2}}} \cdot \frac{dv}{dt} \quad \text{and} \quad a'_N = \frac{v^2}{R \sqrt{1 - \beta^2}}$$

(If R is the curvature of the path in $\mathcal{O}$, the curvature in $\mathcal{O}'$ is $R\sqrt{1 - \beta^2}$.)

The gravitational force is:

$$\begin{aligned} \vec{F}_G &= m_0 \vec{a}' = m_0 (a'_T \vec{e}_T + a'_N \vec{e}_N) \\ &= m_0 \left[\frac{1}{(1 - \beta^2)^{\frac{3}{2}}} \cdot \frac{dv}{dt} \vec{e}_T + \frac{1}{(1 - \beta^2)^{\frac{1}{2}}} \cdot \frac{v^2}{R} \vec{e}_N \right] = \frac{d}{dt} \left[\frac{m_0}{\sqrt{1 - \beta^2}} \cdot \vec{v} \right] \end{aligned}$$

Finally, with: $\frac{m_0}{\sqrt{1-\beta^2}} \cdot \vec{v} = \vec{p}$

We obtain:

$$\vec{F}_G = \frac{d\vec{p}}{dt}$$

$\vec{p}$ is the linear momentum of the point mass relative to the inertial reference frame $\mathcal{O}$. It is the product of its relativistic mass $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

with its velocity $\vec{v}$ in $\mathcal{O}$. The linear momentum of a moving point mass is a measure for its inertia, for its ability to persist in its dynamic state.

5.2.3 The equivalence mass-energy

The instantaneous value of the linear momentum $\vec{p} = m \cdot \vec{v}$ of the point mass m_0 , that freely moves relative to the inertial reference frame $\mathcal{O}$, and the instantaneous value of the force $\vec{F}$ that acts on it, are related by:

$$\vec{F} = \frac{d\vec{p}}{dt}$$

The elementary vectorial displacement $d\vec{r}$ of m_0 during the elementary time interval dt is:

$$d\vec{r} = \vec{v} \cdot dt$$

And the elementary work done by $\vec{F}$ during dt is [8]:

$$dW = \vec{F} \cdot d\vec{r} = \vec{F} \cdot \vec{v} \cdot dt = \vec{v} \cdot d\vec{p}$$

With $\vec{p} = m \cdot \vec{v} = \frac{m_0}{\sqrt{1-(\frac{v}{c})^2}} \cdot \vec{v}$, this becomes:

$$dW = \frac{m_0 \cdot v \cdot dv}{\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{3}{2}}} = d \left[\frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \cdot c^2 \right] = d(m \cdot c^2)$$

The work done on the moving point mass equals, by definition, the increase of the energy of the mass. So, $d(m \cdot c^2)$ is the increase of the energy of the mass and $m \cdot c^2$ is the energy represented by the mass. We can conclude:

A point mass with relativistic mass m is equivalent to an amount of energy of $m \cdot c^2$.

5.2.4 The interaction between two uniform linear moving point masses

5.2.4.1 The interaction between two moving point masses according to S.R.T.

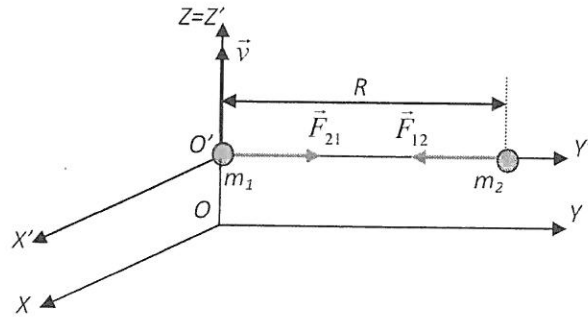


Fig 11

Two particles with rest masses m_1 and m_2 (fig 11) are anchored in the inertial frame O' that is moving relative to the inertial frame O with constant velocity $\vec{v} = v \cdot \vec{e}_y$. The distance between the masses is R .

In $\mathcal{O}'$ the masses are at rest, they don't move. According to Newton's law of universal gravitation, they exert on each other equal but opposite forces:

$$F' = F'_{12} = F'_{21} = G \cdot \frac{m_1 \cdot m_2}{R^2} = \frac{1}{4\pi\eta_0} \cdot \frac{m_1 \cdot m_2}{R^2}$$

In $\mathcal{O}$ both masses are moving with constant speed v in the direction of the Z-axis. From the transformation equations between an inertial frame $\mathcal{O}$ and another inertial frame $\mathcal{O}'$, in which a point mass experiencing a force F' is instantaneously at rest [6], we can immediately deduce the force F that the point masses exert on each other in $\mathcal{O}$:

$$F = F_{12} = F_{21} = F' \cdot \sqrt{1 - \left(\frac{v}{c}\right)^2} = F' \cdot \sqrt{1 - \beta^2}$$

5.2.4.2 The interaction between two moving point masses according to GEM

In §3.5, it is shown that the gravitational field $(\vec{E}_g, \vec{B}_g)$ of a particle with rest mass m_0 that is moving with constant velocity $\vec{v} = v \cdot \vec{e}_z$ along the Z-axis of an inertial frame $\mathcal{O}$ (fig 11) is determined by:

$$\begin{aligned} \vec{E}_g &= -\frac{m_0}{4\pi\eta_0 r^3} \cdot \frac{1 - \beta^2}{(1 - \beta^2 \cdot \sin^2 \theta)^{\frac{3}{2}}} \cdot \vec{r} = -\frac{m_0}{4\pi\eta_0 r^2} \cdot \frac{1 - \beta^2}{(1 - \beta^2 \cdot \sin^2 \theta)^{\frac{3}{2}}} \cdot \vec{e}_r \\ \vec{B}_g &= -\frac{m_0}{4\pi\eta_0 c^2 r^3} \cdot \frac{1 - \beta^2}{(1 - \beta^2 \cdot \sin^2 \theta)^{\frac{3}{2}}} \cdot (\vec{v} \times \vec{r}) \end{aligned}$$

with $\beta = \frac{v}{c}$, the dimensionless speed of m_0 . One can verify that these expressions satisfy the laws of GEM.

In the inertial frame $\mathcal{O}$, the masses m_1 and m_2 are moving in the direction of the Z-axis with speed v . m_2 moves through the GEM field generated by m_1 , and m_1 moves through that generated by m_2 .

According the above formulas, the magnitude of the GEM field created and maintained by m_1 at the position of m_2 is determined by:

$$E_{g2} = \frac{m_1}{4\pi\eta_0 R^2} \cdot \frac{1}{\sqrt{1-\beta^2}} \quad \text{and} \quad B_{g2} = \frac{m_1}{4\pi\eta_0 R^2} \cdot \frac{1}{\sqrt{1-\beta^2}} \cdot \frac{v}{c^2}$$

And according to the force law $\vec{F}_G = m_0 [\vec{E}_g + (\vec{v} \times \vec{B}_g)]$, F_{12} , the magnitude of the force exerted by the gravitational field $(\vec{E}_{g2}, \vec{B}_{g2})$ on m_2 - this is the attraction force of m_1 on m_2 - is: $F_{12} = m_2 \cdot (E_{g2} - v \cdot B_{g2})$

After substitution:
$$F_{12} = \frac{1}{4\pi\eta_0} \cdot \frac{m_1 m_2}{R^2} \cdot \sqrt{1-\beta^2} = F'_{21} \cdot \sqrt{1-\beta^2}$$

In the same way we find:
$$F_{21} = \frac{1}{4\pi\eta_0} \cdot \frac{m_1 m_2}{R^2} \cdot \sqrt{1-\beta^2} = F'_{12} \cdot \sqrt{1-\beta^2}$$

We conclude that the moving masses attract each other with a force:

$$F = F_{12} = F_{21} = F' \cdot \sqrt{1-\beta^2}$$

This result perfectly agrees with that based on S.R.T. (§5.2.4.1). We also can conclude that the component of the gravitational force due to the g-induction is β^2 times smaller than that due to the g-field. This implies that, for speeds much smaller than the speed of light, the effects of the β -information are masked.

The β -information emitted by the rotating sun is not taken into account when the classical theory of gravitation describes the planetary orbits. It can be shown that this is responsible for deviations (as the advance of Mercury Perihelion) of the real orbits with respect to these predicted by that theory [9].

6 The gravitational field of an harmonically oscillating point mass m - Gravito-magnetic waves

In fig 12 we consider a point mass m that harmonically oscillates, with frequency $\nu = \frac{\omega}{2\pi}$, around the origin of the inertial reference frame O . At the moment t it passes at P_1 . We suppose that the speed of the charge is always much smaller than the speed of light and that it is described by:

$$v(t) = V \cdot \cos \omega t$$

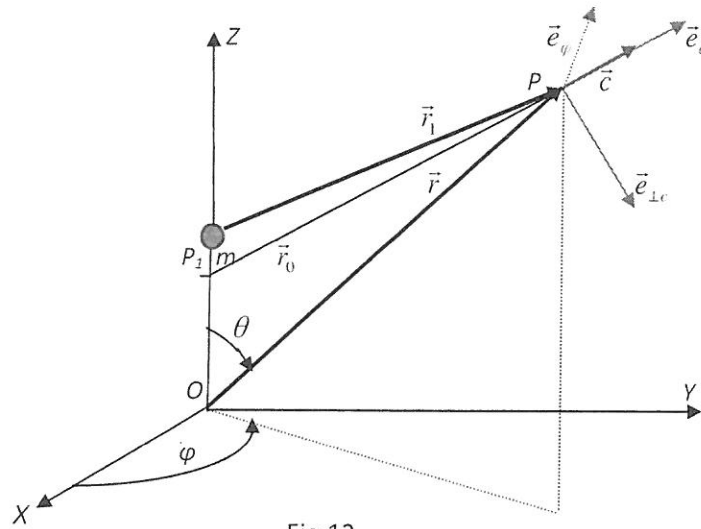


Fig 12

The elongation $z(t)$ and the acceleration $a(t)$ are in this case expressed as:

$$z(t) = \frac{V}{\omega} \cdot \cos(\omega t - \frac{\pi}{2}) \quad \text{and} \quad a(t) = \omega \cdot V \cdot \cos(\omega t + \frac{\pi}{2})$$

We restrict our considerations about the gravitational field of m to points P that are sufficiently far away from the origin O . Under this condition we

can posit that the fluctuation of the length of the vector $\overrightarrow{P_1P} = \vec{r}_1$ is very small relative to the length of the time-independent position vector $\vec{r}$, that defines the position of P relative to the origin O . In other words: we accept that the amplitude of the oscillation is very small relative to the distances between the origin and the points P on which we focus.

6.3.1 The transversal gravitational field of an harmonically oscillating point mass

Starting from $\bar{V} = V.e^{j\omega t}$ - the complex quantity representing $v(t)$ - the complex representation $\bar{E}_{g\perp c}$ of the time dependant part of the transversal component of $\vec{E}_g$ and the complex representation $\bar{B}_{gq'}$ of the time dependant part of $\vec{B}_g$ in P follow immediately from §3.8.2:

$$\bar{E}_{g\perp c} = -\frac{m\bar{V}}{4\pi} \cdot e^{-j.k.r} \cdot \left(\frac{1}{\eta_0 \cdot c \cdot r^2} + \frac{j.\omega.V_0}{r} \right) \cdot \sin \theta$$

$$\bar{B}_{gq'} = -\frac{v_0 \cdot m \cdot \bar{V}}{4\pi} \cdot e^{-j.k.r} \cdot \left(\frac{1}{r^2} + \frac{j.k}{r} \right) \cdot \sin \theta$$

Where $k = \frac{\omega}{c}$ the phase constant. Note that $\bar{B}_{gq'} = \frac{\bar{E}_{g\perp c}}{c}$.

So, an harmonically oscillating point mass emits a transversal "gravitomagnetic" wave that - relative to the position of the mass - expands with the speed of light:

$$B_{gq'}(r, \theta; t) = \frac{E_{g\perp c}(r, \theta; t)}{c} = \frac{v_0 \cdot m \cdot V \cdot \sin \theta \cdot \sqrt{1 + k^2 r^2}}{4\pi \cdot c^2} \cdot \cos(\omega t - kr + \Phi + \pi)$$

with $tg\Phi = kr$.

In points at a great distance from the oscillating charge, specifically there where $r \gg \frac{1}{k} = \frac{c}{\omega}$, this expression asymptotically equals:

$$B_{g\psi} = \frac{E_{\perp c}}{c} = \frac{\nu_0 \cdot k \cdot m \cdot V \cdot \sin \theta}{4\pi r} \cdot \sin(\omega t - kr) = \frac{\nu_0 \cdot m \cdot \omega \cdot V \cdot \sin \theta}{4\pi c r} \cdot \sin(\omega t - kr)$$

$$= - \frac{\nu_0 \cdot m \cdot a(t - \frac{r}{c}) \cdot \sin \theta}{4\pi c r}$$

The intensity of the "far field" is inversely proportional to r , and is determined by the component of the acceleration of m , that is perpendicular to the direction of $\vec{e}_c$.

6.3.2 The longitudinal gravitational field of an harmonically oscillating point mass

The oscillation of the point mass m along the Z -axis is responsible for the existence of a fluctuation of $r_0 = P_1 P$ (fig 12), the distance travelled by the informations that at the moment t pass near P . Within the framework of our approximations:

$$r_0(t) \approx r - z(t - \frac{r}{c}) \cdot \cos \theta = r \cdot \left\{ 1 - \frac{z(t - \frac{r}{c})}{r} \cdot \cos \theta \right\}$$

and

$$\left(\frac{1}{r_0}\right)^2 \approx \frac{1}{r^2} \cdot \left(1 + 2 \cdot \frac{z(t - \frac{r}{c})}{r} \cdot \cos \theta\right)$$

From §3.8.2, it follows: $E_{gc} = -\frac{m}{4\pi \cdot \eta_0 \cdot r^2} - \frac{m}{4\pi \cdot \eta_0 \cdot r^3} \cdot 2 \cdot z(t - \frac{r}{c}) \cdot \cos \theta$

So $\bar{E}_{gc}$, the complex representation of the time dependant part of the longitudinal gravitational field is:

$$\bar{E}_{gc} = -\frac{m \cdot \bar{V}}{4\pi} \cdot e^{-jkr} \cdot \frac{2}{j \cdot \omega \cdot \eta_0 \cdot r^3} \cdot \cos \theta$$

We conclude that an harmonically oscillating point mass emits a longitudinal gravitational wave that - relative to the position of the mass - expands with the speed of light:

$$E_{gc}(r, \theta; t) = \frac{m.V}{4\pi.\eta_0.c.k} \cdot \frac{2}{r^3} \cdot \sin(\omega t - kr)$$

6.3.3 The energy radiated by an harmonically oscillating point mass

6.3.3.1 Poynting's theorem

In empty space a gravitational field is completely defined by the vectorial functions $\vec{E}_g(x, y, z; t)$ and $\vec{B}_g(x, y, z; t)$. It can be shown that the spatial area G enclosed by the surface S - at the moment t - contains an amount of energy given by the expression:

$$U = \iiint_G \left(\frac{\eta_0 \cdot E_g^2}{2} + \frac{B_g^2}{2\nu_0} \right) \cdot dV$$

The rate at which the energy escapes from G is:

$$-\frac{\partial U}{\partial t} = -\iiint_V \left(\eta_0 \cdot \vec{E}_g \cdot \frac{\partial \vec{E}_g}{\partial t} + \frac{1}{\nu_0} \cdot \vec{B}_g \cdot \frac{\partial \vec{B}_g}{\partial t} \right) \cdot dV$$

According to the third law of GEM: $rot \vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$,

and according to the fourth law: $rot \frac{\vec{B}_g}{\nu_0} = \eta_0 \cdot \frac{\partial \vec{E}_g}{\partial t}$. So:

$$-\frac{\partial U}{\partial t} = \iiint_G \left(\frac{\vec{B}_g}{\nu_0} \cdot rot \vec{E}_g - \vec{E}_g \cdot rot \frac{\vec{B}_g}{\nu_0} \right) \cdot dV = \iiint_G div \left(\frac{\vec{E}_g \times \vec{B}_g}{\nu_0} \right) \cdot dV$$

By application of the theorem of Ostrogradsky [5]: $\iiint_G \text{div} \vec{F} . dV = \oiint_S \vec{F} . \vec{dS}$,

one can rewrite this as: $-\frac{\partial U}{\partial t} = \oiint_S \frac{\vec{E}_g \times \vec{B}_g}{\nu_0} . \vec{dS}$ from which one can

conclude that the expression $\frac{\vec{E}_g \times \vec{B}_g}{\nu_0} . \vec{dS}$ defines the rate at which energy flows through the surface element dS in P in the sense of the positive normal.

So, the density of the energy flow in P is: $\frac{\vec{E}_g \times \vec{B}_g}{\nu_0}$. This vectorial quantity is called the "Poynting's vector". It is represented by $\vec{P}$:

$$\vec{P} = \frac{\vec{E}_g \times \vec{B}_g}{\nu_0}$$

The amount of energy transported through the surface element dS in the sense of the positive normal during the time interval dt is:

$$dU = \frac{\vec{E}_g \times \vec{B}_g}{\nu_0} . \vec{dS} . dt$$

6.3.3 The energy radiated by an harmonically oscillating point mass - gravitons

Under §6.3.1 it is shown that an harmonically oscillating point mass m radiates a gravitomagnetic wave that at a far point P is defined by (see fig 12):

$$\vec{E} = E_{\perp c} . \vec{e}_{\perp c} = \frac{\nu_0 . m . \omega . V . \sin \theta}{4\pi r} . \sin(\omega t - kr) . \vec{e}_{\perp c}$$

$$\vec{B} = B_{\varphi} . \vec{e}_{\varphi} = \frac{\nu_0 . m . \omega . V . \sin \theta}{4\pi cr} . \sin(\omega t - kr) . \vec{e}_{\varphi}$$

The instantaneous value of Poynting's vector in P is:

$$\vec{P} = \frac{\nu_0 . m^2 . \omega^2 . V^2 . \sin^2 \theta}{16 \pi^2 . c . r^2} . \sin^2 (\omega t - kr) . \vec{e}_c$$

The amount of energy that, during one period T , flows through the surface element dS that at P is perpendicular to the direction of the movement of the informatoms, is:

$$dU = \int_0^T P . dt . dS = \frac{\nu_0 . m^2 . \omega^2 . V^2 . \sin^2 \theta}{16 \pi^2 . c . r^2} . \frac{T}{2} . dS$$

And, with $\omega = \frac{2\pi}{T} = 2\pi . \nu$:

$$dU = \frac{\nu_0 . m^2 . V^2 . \sin^2 \theta}{8c} . \nu . \frac{dS}{r^2}$$

$\frac{dS}{r^2} = d\Omega$ is the solid angle under which dS is "seen" from the origin. So, the oscillating mass radiates per unit of solid angle in the direction θ , per period, an amount of energy u_Ω :

$$u_\Omega = \frac{\nu_0 . m^2 . V^2 . \sin^2 \theta}{8c} . \nu$$

The density of the flux of energy is greatest in the direction perpendicular to the movement of the mass ($\theta = 90^\circ$). Further, the radiated energy is proportional to the frequency of the wave, thus proportional to the frequency at which the mass oscillates.

We posit that an oscillating mass m loads some of the informatoms that it emits with a discrete energy packet ($h' . \nu$).

h' plays the role of Planck's constant in electromagnetism, but his value depends on the nature of the emitter. Informatoms carrying an energy packet are called "gravitons". In other words, we postulate that the gravitomagnetic energy radiated by an oscillating point mass is

transported by informatons. This implies that gravitons rush through space with the speed of light.

6.3.3.3 Estimate of the value of h'

The number of gravitons emitted by an oscillating point mass m per period and per unit of solid angle in the direction θ is:

$$N_{g\Omega} = \frac{u_{\Omega}}{h' \cdot \nu} = \frac{\nu_0 \cdot m^2 \cdot V^2 \cdot \sin^2 \theta}{8 \cdot h' \cdot c}$$

It follows that the total number of gravitons that are emitted per period is:

$$N_g = \frac{\nu_0 \cdot m^2 \cdot V^2}{8 \cdot h' \cdot c} \cdot 2\pi \cdot \int_0^{\pi} \sin^3 \theta \cdot d\theta = \frac{\pi}{3} \cdot \frac{\nu_0}{h' \cdot c} \cdot m^2 \cdot V^2$$

If the oscillating point mass is an electron:

$$N_g = 2,71 \cdot 10^{-95} \frac{V^2}{h'_e}$$

An oscillating electron also emits photons. The number of photons it emits per period is [11]:

$$N_f = 1,70 \cdot 10^{-19} \cdot V^2$$

If we assume that the number of emitted gravitons equals the number of emitted photons:

$$N_f = 1,70 \cdot 10^{-19} \cdot V^2 = 2,71 \cdot 10^{-95} \cdot \frac{V^2}{h'_e} = N_g$$

From which follows: $h'_e = 1,60 \cdot 10^{-76} \text{ J} \cdot \text{s}$

For an oscillating proton, an analogue reasoning leads to:

$$h'_p = 5,35 \cdot 10^{-70} \text{ J} \cdot \text{s}$$

7 THE NATURE OF THE GRAVITATIONAL FIELD

1. According to the postulate of the emission of informatons, the gravitational field of a mass at rest is characterized by the following statements:

01. *Gravitational phenomena propagate with the speed of light.*
02. *The gravitational field is granular.*
03. *The gravitational field continuously regenerates.*
04. *The gravitational field shows fluctuations.*
05. *The gravitational field expands with the speed of light.*
06. *In a gravitational field, there is conservation of g-information, what mathematically can be expressed as a relation between the spatial variation of the g-field $\vec{E}_g$ and the mass density ρ_G in any*

$$\text{point of the field: } \operatorname{div} \vec{E}_g = -\frac{\rho_G}{\eta_0}.$$

2. Complementary, the following statements are valid for the gravitational field of a uniformly moving mass.

07. *The g-field $\vec{E}_g$ of a point mass that is moving with constant velocity always points to the actual position of that mass.*
08. *The g-induction $\vec{B}_g$ shows fluctuations.*
09. *From the definition of the β -index, it follows: $\operatorname{div} \vec{B}_g = 0$.*
10. *The spatial variation of the g-induction in a point of a gravitational field depends on the density of the mass flow $\vec{J}_G$*

$$\text{in that point: } \operatorname{rot} \vec{B}_g = -v_0 \cdot \vec{J}_G \quad \text{with } v_0 = \frac{1}{\eta_0 \cdot c^2}.$$

3. The definitions of $\vec{E}_g$ and of $\vec{B}_g$ can be extended to the situation where the gravitational field is generated by a set of whether or not - uniformly or not uniformly - moving point masses or by a whether or not moving mass continuum. In that general case, the statements 1 - 10 stay valid. In addition, it follows from the kinematics of an informaton that in empty space:

$$11.1. \operatorname{rot} \vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t} \quad \text{and} \quad 11.2. \operatorname{rot} \vec{B}_g = \frac{1}{c^2} \cdot \frac{\partial \vec{E}_g}{\partial t}$$

12. *There is a perfect isomorphism between the gravitational field and the electromagnetic field: at a point P situated in a mass continuum that is characterized by the mass density ρ_G and the mass flow density $\vec{J}_G$, $\vec{E}_g$ and $\vec{B}_g$ satisfy the following equations:*

$$12.1. \operatorname{div} \vec{E}_g = -\frac{\rho_G}{\eta_0} \quad 12.3. \operatorname{rot} \vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$$

$$12.2. \operatorname{div} \vec{B}_g = 0 \quad 12.4. \operatorname{rot} \vec{B}_g = -\nu_0 \cdot \vec{J}_G + \frac{1}{c^2} \cdot \frac{\partial \vec{E}_g}{\partial t}$$

4. In an inertial reference system, the gravitational interaction between masses is determined by the force law of GEM that is analogue to the force law of EM. It is the expression of the fact that a point mass tends to become blind for the flow of information generated by other point masses.

13. *A point mass with rest mass m_0 that moves with velocity $\vec{v}$ through a gravitational field $(\vec{E}_g, \vec{B}_g)$ experiences a force*

$$\vec{F}_G = m_0 \cdot [\vec{E}_g + (\vec{v} \times \vec{B}_g)]$$

5. An accelerated mass is the source of a gravito-magnetic wave.

14. *An oscillating point mass radiates a gravito-magnetic wave, transporting energy in the form of granular energy packets called "gravitons".*

EPILOGUE

1. The theory of informatons is also able to explain the phenomena and the laws of electromagnetism [7][10]. It is sufficient to add the following rule to the postulate of the emission of informatons:

Informatons emitted by an electrically charged point mass (a "point charge" q) at rest in an inertial reference frame, carry an attribute referring to the charge of the emitter, namely the e -index. e -indices are represented as $\vec{s}_e$ and defined by:

1. The e -indices are radial relative to the position of the emitter. They are centrifugal when the emitter carries a positive charge ($q = +Q$) and centripetal when the charge of the emitter is negative ($q = -Q$).
2. s_e , the magnitude of an e -index depends on Q/m , the charge per unit of mass of the emitter. It is defined by:

$$s_e = \frac{1}{K \cdot \epsilon_0} \cdot \frac{Q}{m} = 8,32 \cdot 10^{-40} \cdot \frac{Q}{m} N \cdot m^2 \cdot s \cdot C^{-1}$$

where $\epsilon_0 = 8,85 \cdot 10^{-12} F/m$ is the permittivity constant.

Consequently (cfr §3), the informatons emitted by a moving point charge q have at the fixed point P - defined by the time dependant position vector $\vec{r}$ (cfr fig 5) - two attributes that are in relation with the fact that q is a moving point charge: their e -index $\vec{s}_e$ and their b -index $\vec{s}_b$:

$$\vec{s}_e = \frac{q}{m} \cdot \frac{1}{K \cdot \epsilon_0} \cdot \vec{e}_r = \frac{q}{m} \cdot \frac{1}{K \cdot \epsilon_0} \cdot \frac{\vec{r}}{r} \quad \text{and} \quad \vec{s}_b = \frac{\vec{c} \times \vec{s}_e}{c} = \frac{\vec{v} \times \vec{s}_e}{c}$$

Macroscopically, these attributes manifest themselves at P as, respectively the *electric field strength* (the e -field) $\vec{E}$ and the *magnetic induction* (the b -induction) $\vec{B}$.

2. The assumption that a photon is nothing else than an informaton transporting an energy package can explain the duality of light [7][10][11].
3. The fact that the "theory of informatons" permits to understand the nature of gravitation and to deduce the laws that govern the gravitational phenomena justifies the hypothesis that "information" is the substance of the gravitational field and it supports the idea that informatons really exist.

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**ON THE INTERPRETATION OF THE LORENTZ
TIME TRANSFORMATION**

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Abstract

Einstein gave two conflicting interpretations of the Lorentz transformation for time, $\tau = t\sqrt{1-v^2/c^2}$, applied to a moving clock. The first was as a coordinate transformation, which was the basis of its derivation. The second was as a physical slowing effect on the moving clock caused solely by its motion relative to a stationary reference clock. These interpretations are not independent. That is, the Lorentz coordinate transformation cannot be applied during the clock's time of motion without correcting for the lack of synchronization between the moving and stationary clocks resulting from the slowing of the moving clock. Otherwise, the Lorentz transformation gives an incorrect result. In addition, the interpretation as a physical effect has seemingly insurmountable logical difficulties, as it subjects the moving clock to a physical slowing dependent upon an arbitrary inertial reference frame, and which is therefore indeterminable. This interpretation is supported by questionable experimental evidence.

Keywords: Lorentz transformations; Time transformations; Relativity; Special Relativity; Coordinate transformations; Time dilation; Twin paradox.

After deriving the Lorentz transformations as coordinate transformations, Einstein [1], in section 4, "Physical Meaning of the Equations Obtained in Respect to Moving Rigid Bodies and Moving Clocks", makes the rather astonishing claim that the Lorentz time coordinate transformation,

$$\tau = t\sqrt{1-v^2/c^2} = t - (1 - \sqrt{1-v^2/c^2})t, \quad (1)$$

represents a physical slowing effect on a moving clock due merely to its motion relative to a stationary reference clock.

This interpretation not only has no apparent basis in his derivation, but also contradicts it. Now the clocks are no longer physically synchronized at $t > 0$, so the Lorentz transformation (1) is no longer valid during the time of motion. By Einstein's argument, the time interval of the moving clock measured in its own coordinate system at $t > 0$ is no longer τ , but is decreased by the amount

$$(1 - \sqrt{1-v^2/c^2})t.$$

That is, the clocks are synchronized at $t = \tau = 0$, but are out of synchronization at $t > 0$ by this amount, so that the time coordinate measured by the moving clock at $t > 0$ in its own coordinate system is not τ , but rather

$$\tau' = \tau - (1 - \sqrt{1-v^2/c^2})t. \quad (2)$$

The correct transformation at $t > 0$ then becomes

$$\tau' = (t - (1 - \sqrt{1 - v^2/c^2})t) - (1 - \sqrt{1 - v^2/c^2})t, \text{ or}$$

$$\tau' = t - 2(1 - \sqrt{1 - v^2/c^2})t, \quad (3)$$

which gives a slowing during the time of motion twice that of the Lorentz transformation (1).

But even then, the interpretation as a physical effect seems to me impossible, because the slowing depends upon an arbitrary choice of inertial reference frame of the stationary clock, and which therefore cannot be uniquely determined. That is, an arbitrary number of inertial reference frames different from the moving clock would subject the clock to an arbitrary number of different slowing effects at the same time, or if we limit the effect to one stationary reference frame, then by simply choosing either of two different stationary reference frames we would produce two different results.

The experimental evidence claiming to support this second interpretation can be divided into two types. The first measures the clock during the time of motion. In this case, if the observed time difference appears to be consistent with the Lorentz transformation, then the experiment would, according to the above argument, support the first interpretation, but not the second. Therefore this type of experiment need not be examined.

The second type of experiment is one where the moving clock is brought to rest and compared with the stationary clock, the elapsed time interval of motion compared, so that only a physical change over the time of travel would be observed. The results claimed by [2] are based on the behavior of synchronized cesium clocks on the Earth's surface and in aircraft flying in opposite directions with respect to the Earth's rotation, and then returning to the point of departure.

The elapsed times of the different clocks over the time of travel were compared with the predicted effect from the Lorentz transformation using

the Earth's axis as the stationary reference frame. Clocks flown in planes going in one direction showed a slower rate of time measure during the motion than the earth-bound clock, and those in the planes flying the opposite direction showed an increased rate. The experiment was criticized by [3,4] and was claimed to have been confirmed by [5].

I find the results interesting, as none of the issues discussed here are mentioned. In particular no reason is given why the clocks experienced the effects from the particular choice of stationary reference frame of the Earth's axis, but not from some other, in which case the clocks would have shown different results. An obvious choice would be the reference frame of the ground-based clock, that of the first example given by Einstein. In that case, both traveling clocks would have shown a slowing with respect to the ground-based clock.

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**DYNAMIC ISO-TOPIC LIFTING WITH APPLICATION TO
FIBONACCI'S SEQUENCE AND MANDELBROT'S SET**

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Abstract

In this exploration, we introduce and define “dynamic iso-spaces”, which are cutting-edge iso-mathematical constructions that are built with “dynamic iso-topic liftings” for “dynamic iso-unit functions”. For this, we consider both the continuous and discrete cases. Subsequently, we engineer two simple examples that engage Fibonacci's sequence and Mandelbrot's set to define a “Fibonacci dynamic iso-space” and a “Mandelbrot dynamic iso-space”, respectively. In total, this array of resulting iso-structures indicates that a new branch of iso-mathematics may be in order.

Keywords: Geometry and topology; Santilli iso-number; Santilli iso-space; Dynamic iso-space; Fractal geometry; Fibonacci sequence; Mandelbrot set.

1 Introduction

A *gigantic* problem of pure mathematics, in the context of number theory, is to establish a *universal* number classification system for describing the physical laws of nature, such as the identification of all sets that occur with numeric field axioms. During his relentless pursuit of this problem, scientific pioneer R.M. Santilli successfully circumvented the original five number field axioms [1] to create the new field of iso-mathematics [2, 3, 4, 5], which became the foundation of hadronic mechanics [6].

Since its initiation, Santilli's iso-mathematics [2, 3, 4, 5] has continued to gain momentum in application and popularity, but this emergent discipline is—to say the least—*vast*, relatively adolescent, and is currently not considered to be “mainstream” in the community of science, technology, engineering, and mathematics. Therefore, the iso-mathematics realm remains largely unexplored, with unknown bounds on its utilization potential.

In this paper, we venture into iso-mathematics [2, 3, 4, 5] by considering iso-spaces that systematically *change*. In Section 2, we introduce the notion of *dynamic iso-topic liftings* by assembling definitions for *continuous dynamic iso-spaces* and *discrete dynamic iso-spaces*. Afterwards, we give two examples that demonstrate how dynamic iso-spaces can be lifted for fractal structures in Section 3. Finally, we conclude our exploration with Section 4, where we recapitulate the results with brief discussion and outlook on future modes of research.

2 Dynamic iso-spaces

Here, we introduce and define the dynamic iso-space with its dynamic iso-topic lifting. For this, we follow Santilli's methodology [2, 3, 4, 5] and consider the following procedure:

1. Let S be a space.
2. Let $S_{\hat{t}}$ be an iso-space that results from the iso-topic lifting $S \rightarrow S_{\hat{t}}$ for the iso-unit $\hat{t} > 0$ with the corresponding inverse $\hat{\tau} = \frac{1}{\hat{t}} > 0$, such that

$$\begin{aligned} f(\hat{t}) : S &\rightarrow S_{\hat{t}} \\ f^{-1}(\hat{t}) : S_{\hat{t}} &\rightarrow S, \end{aligned} \tag{1}$$

so clearly S and $S_{\hat{t}}$ are locally iso-morphic.

3. **Dynamic iso-space.** Now, we wish to show that $S_{\hat{t}}$ is an iso-space that is characterized by constant change, meaning that $S_{\hat{t}}$ is a dynamic iso-space. Hence, we will show that $S_{\hat{t}}$ varies in accordance to some function; here, this positive-definite function will serve as the *dynamic iso-unit* that varies because its parameter varies.
4. Thus, let $r > 0$ be the parameter that varies as $r \rightarrow \infty$ for the *generic form* of the *dynamic iso-unit function*

$$\hat{t} \equiv \hat{\delta}(r) > 0, \quad r \rightarrow \infty, \quad (2)$$

to consequently define the dynamic iso-space

$$S_{\hat{t}} \equiv S_{\hat{\delta}(r)}. \quad (3)$$

5. **Continuous dynamic iso-space.** First, we will show that the $S_{\hat{\delta}(r)}$ of eq. (3) can be rewritten and defined as a *continuous* dynamic iso-space if its corresponding dynamic iso-unit function is continuous as its parameter r varies; for this, the constraint is that r must take on the values of an infinite continuous sequence of positive-definite numbers. Hence, for example, if $\mathbb{R}^+$ is the set of all positive-definite real numbers, then let $r \in \mathbb{R}^+$ be the positive-definite, real-valued, varying parameter for the *continuous dynamic iso-unit function*

$$\hat{t} \equiv \hat{\delta}_{\mathbb{R}^+}(r) \equiv 2\pi r > 0, \quad r \in \mathbb{R}^+, \quad r \rightarrow \infty_{\mathbb{R}^+}, \quad (4)$$

to consequently define the continuous dynamic iso-space

$$S_{\hat{t}} \equiv S_{\hat{\delta}_{\mathbb{R}^+}(r)}, \quad (5)$$

where we rewrite eq. (1) to encompass the continuous dynamic structure of eqs. (4-5) as

$$\begin{aligned} f(\hat{\delta}_{\mathbb{R}^+}(r)) : S &\rightarrow S_{\hat{\delta}_{\mathbb{R}^+}(r)} \\ f^{-1}(\hat{\delta}_{\mathbb{R}^+}(r)) : S_{\hat{\delta}_{\mathbb{R}^+}(r)} &\rightarrow S \end{aligned} \quad (6)$$

so S and $S_{\hat{\delta}_{\mathbb{R}^+}(r)}$ remain locally iso-morphic as $r \rightarrow \infty_{\mathbb{R}^+}$. Note that the 2π circumference scaling of eq. (4) is arbitrary for this example, as we can make eq. (4) anything we want as long as the resulting iso-unit $\hat{t}$ is positive-definite and varies continuously as $r \rightarrow \infty_{\mathbb{R}^+}$.

6. **Discrete dynamic iso-space.** Second, we will show that the $S_{\hat{\delta}(r)}$ of eq. (3) can be rewritten and defined as a *discrete* dynamic iso-space if its corresponding dynamic iso-unit function is discrete as its parameter r varies; for this, the constraint is that r must take on the values of an infinite discrete sequence of positive-definite numbers. Hence, for example, if $\mathbb{N}$ is the set of all natural numbers (positive integers), then let $r \in \mathbb{N}$ be the positive-definite, natural-valued, varying parameter for the *discrete dynamic iso-unit function*

$$\hat{t} \equiv \hat{\delta}_{\mathbb{N}}(r) \equiv 2\pi r > 0, \quad r \in \mathbb{N}, \quad r \rightarrow \infty_{\mathbb{N}}, \quad (7)$$

to consequently define the discrete dynamic iso-space

$$S_{\hat{t}} \equiv S_{\hat{\delta}_{\mathbb{N}}(r)}, \quad (8)$$

where we rewrite eq. (1) to encompass the discrete dynamic structure of eqs. (7-8) as

$$\begin{aligned} f(\hat{\delta}_{\mathbb{N}}(r)) : S &\rightarrow S_{\hat{\delta}_{\mathbb{N}}(r)} \\ f^{-1}(\hat{\delta}_{\mathbb{N}}(r)) : S_{\hat{\delta}_{\mathbb{N}}(r)} &\rightarrow S \end{aligned} \quad (9)$$

so S and $S_{\hat{\delta}_{\mathbb{N}}(r)}$ remain locally iso-morphic as $r \rightarrow \infty_{\mathbb{N}}$. Again, note that the 2π circumference scaling of eq. (7) is arbitrary for this example, as we can make eq. (7) anything we want as long as the resulting iso-unit $\hat{t}$ is positive-definite and varies discretely as $r \rightarrow \infty_{\mathbb{N}}$.

At this point, the results of eqs. (1-9) indicate the existence of a new family of iso-spaces called dynamic iso-spaces, where a member of this family (continuously or discretely) deforms in accordance to Santilli's topological-preservations as r varies. Hence, we establish the following:

Lemma 1. *A space S is locally iso-morphic to the dynamic iso-space $S_{\hat{\delta}(r)}$ under the topologically-preserving dynamic iso-topic lifting $S \rightarrow S_{\hat{\delta}(r)}$ for the dynamic iso-unit function $\hat{\delta}(r) > 0$ with inverse $\hat{\delta}^{-1}(r) = \frac{1}{\hat{\delta}(r)} > 0$, where the positive-definite parameter r varies indefinitely.*

Lemma 2. *A dynamic iso-space $S_{\hat{\delta}(r)}$ is a continuous dynamic iso-space if the dynamic iso-unit function $\hat{\delta}(r)$ is continuous as its parameter r varies.*

Lemma 3. *A dynamic iso-space $S_{\hat{\delta}(r)}$ is a discrete dynamic iso-space if the dynamic iso-unit function $\hat{\delta}(r)$ is discrete as its parameter r varies.*

3 Application Examples

Here, we provide two short examples that engage the dynamic iso-space concept. For this, we opt that both applications include fractals.

3.1 Fibonacci dynamic iso-space

First, we introduce and define a Fibonacci dynamic iso-space, which is a specific type of discrete dynamic iso-space. The well-known Fibonacci sequence is [7]

$$F = \{0, 1, 1, 2, 3, 5, 8, \dots\}. \quad (10)$$

Hence, we identify the n th Fibonacci number in the sequence F with the “Fibonacci function” $F(n)$, which returns the initial values

$$\begin{aligned} F(0) = 0, \quad F(1) = 1, \quad F(2) = 1, \quad F(3) = 2, \quad F(4) = 3, \\ F(5) = 5, \quad F(6) = 8, \quad \dots \end{aligned} \quad (11)$$

Now suppose that we begin with the positive-definite $r = F(1)$ and let $r \in F - \{0\}$ discretely varies or “jumps” according to Fibonacci’s eqs. (10–11)—so in this Fibonacci case, as $r \rightarrow \infty_{Fib}$, r takes on only Fibonacci numbers (excluding zero), which satisfies the discrete iso-unit constraints that $F - \{0\}$ must be an infinite discrete sequence positive-definite numbers. Thus, the discrete dynamic iso-unit function of eq. (7) can be rewritten to define the *Fibonacci dynamic iso-unit function* as

$$\hat{t} \equiv \hat{\delta}_{Fib}(r) \equiv 2\pi r > 0, \quad r \in F - \{0\}, \quad r \rightarrow \infty_{Fib}, \quad (12)$$

to consequently define the Fibonacci dynamic iso-space

$$S_{\hat{t}} \equiv S_{\hat{\delta}_{Fib}(r)} \quad (13)$$

with the Fibonacci dynamic iso-topic liftings

$$\begin{aligned} f(\hat{\delta}_{Fib}(r)) : \quad S &\rightarrow S_{\hat{\delta}_{Fib}(r)} \\ f^{-1}(\hat{\delta}_{Fib}(r)) : \quad S_{\hat{\delta}_{Fib}(r)} &\rightarrow S. \end{aligned} \quad (14)$$

Here, we observe that $\hat{t}$ strictly acquires “Fibonacci circumference” values for the dynamic lifting because r is always treated as a “Fibonacci radius”, which indicates that $S_{\delta_{Fib}(r)}$ iteratively deforms as $r \rightarrow \infty_{Fib}$, where S and $S_{\delta_{Fib}(r)}$ remain locally iso-morphic.

At this point, we’ve completed our first fractal example because we’ve successfully defined a Fibonacci dynamic iso-space.

3.2 Mandelbrot dynamic iso-space

Second, we introduce and define a Mandelbrot dynamic iso-space, which is a specific type of discrete dynamic iso-space because we obtain each value, one-by-one, as we systematically generate Mandelbrot’s set by iterating his complex quadratic polynomial [8, 9]

$$z_{n+1} = z_n^2 + c, \quad (15)$$

where $z_n, z_{n+1}, c \in \mathbb{C}$ are complex numbers. Thus, imagine that we select some c and z_0 and continuously iterate eq. (15) to obtain the discrete Mandelbrot sequence

$$M = \{z_0, z_1, z_2, z_3, z_4, z_5, \dots\}. \quad (16)$$

Hence, similarly to the Fibonacci case of eq. (11), we identify the n th Mandelbrot number in the sequence M with the “Mandelbrot function” $M(n)$ that depends on c , which returns the initial values

$$\begin{aligned} M(0) &= z_0, \quad M(1) = z_1, \quad M(2) = z_2, \quad M(3) = z_3, \quad M(4) = z_4, \\ M(5) &= z_5, \quad \dots \end{aligned} \quad (17)$$

Now we need positive-definite values for our dynamic iso-units, so how do we translate the *complex-valued* (and possibly negative-valued) results of Mandelbrot’s eq. (17) as we did to Fibonacci’s eq. (11)? Well, there are multiple ways to approach this rich problem, so honestly at this point it is a matter of preference. Thus, for the sake of example simplicity, we opt to use the real-valued modulus

$$|z_n| = \sqrt{z_{n_R}^2 + z_{n_I}^2} \quad (18)$$

for the n th iterate of eq. (17) because it is positive-definite. Consequently, we list the moduli of eqs. (16) and (17) as

$$M_{Mod} = \{|z_0|, |z_1|, |z_2|, |z_3|, |z_4|, |z_5|, \dots\} \quad (19)$$

and

$$\begin{aligned} M_{Mod}(0) &= |z_0|, \quad M_{Mod}(1) = |z_1|, \quad M_{Mod}(2) = |z_2|, \quad M_{Mod}(3) = |z_3|, \\ M_{Mod}(4) &= |z_4|, \quad M_{Mod}(5) = |z_5|, \quad \dots, \end{aligned} \quad (20)$$

respectively.

Next, suppose that we begin with the positive-definite $r = M_{Mod}(0)$ and let $r \in M_{Mod}$ discretely vary according to Mandelbrot's moduli of eqs. (19–20)—so in this selected Mandelbrot case, as $r \rightarrow \infty_{Man}$, r takes on only the moduli of Mandelbrot numbers, which satisfies the discrete iso-unit constraints that M_{Mod} must be an infinite discrete sequence of positive-definite numbers. Thus, the discrete dynamic iso-unit function of eq. (7) can be rewritten to define the *Mandelbrot dynamic iso-unit function* as

$$\hat{t} \equiv \hat{\delta}_{Man}(r) \equiv 2\pi r > 0, \quad r \in M_{Mod}, \quad r \rightarrow \infty_{Man}, \quad (21)$$

to consequently define the Mandelbrot dynamic iso-space

$$S_{\hat{t}} \equiv S_{\hat{\delta}_{Man}(r)} \quad (22)$$

with the Mandelbrot dynamic iso-topic liftings

$$\begin{aligned} f(\hat{\delta}_{Man}(r)) : \quad S &\rightarrow S_{\hat{\delta}_{Man}(r)} \\ f^{-1}(\hat{\delta}_{Man}(r)) : \quad S_{\hat{\delta}_{Man}(r)} &\rightarrow S. \end{aligned} \quad (23)$$

Here, we observe that $\hat{t}$ strictly acquires “Mandelbrot circumference” values for the dynamic lifting because r is always treated as a “Mandelbrot radius”, which indicates that $S_{\hat{\delta}_{Man}(r)}$ iteratively deforms as $r \rightarrow \infty_{Man}$, where S and $S_{\hat{\delta}_{Man}(r)}$ remain locally iso-morphic.

At this point, we've completed our second fractal example because we've successfully defined one version of a Mandelbrot dynamic iso-space.

4 Conclusion

The results of this work include original definitions and lemmas for continuous and discrete dynamic iso-spaces, and example constructions of Fibonacci and Mandelbrot dynamic iso-spaces. These dynamic iso-spaces may serve as important tools for future analysis and exploration. But there is still much work to do, as we must continue to relentlessly scrutinize, challenge, and upgrade this emerging framework via the scientific method. In particular, we suggest that in order to test the validity of our results and advance the general capability and applicability of these dynamic iso-systems to subsequent levels, a thorough and rigorous iso-mathematical investigation should be conducted along this research trajectory. For this, we must prove the said lemmas and expand the framework by instantiating additional pertinent families of dynamic iso-structures, and furthermore the dynamic geno-structures, dynamic hyper-structures, and dynamic iso-dual-structures.

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THE MISSING MASS OF THE MILKY WAY GALAXY

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Abstract

This work verifies the principle that gravitation is caused by *energy* and not simply by mass alone. A model is proposed in which cosmic ray protons flow radially through the galaxy. The resulting electric field energy creates a gravitational force, in addition to the conventional Newtonian force. The model yields a rotation curve that agrees in detail with the experimental curve. The total electric field energy is calculated. It is the missing mass of the galaxy.

1. Introduction.

In the proposed model, electrons are separated from protons by the galactic magnetic field. Similar charge separations are known to occur in our solar system. For example, the solar wind is a plasma with enough thermal energy to escape the Sun. The flux is neutral, composed almost equally of protons and electrons (there are some ions, as well). The total flux is 10^{36} particles sec^{-1} , moving with an average velocity of 400 km sec^{-1} . However, the lighter electrons have the greater outward velocity, and a charge separation occurs. This gives rise to an electric field which aids the acceleration of ions. [1]

A second example is provided by cosmic rays, which are very energetic charged particles. The cosmic ray flux arriving at Earth is not neutral: protons make up 90% of the total. It is thought that magnetic fields in the solar system deflect the electrons, thereby causing the observed separation of charge.

The central bulge of the Milky Way Galaxy contains some 10^{10} stars. Assuming a thermal flux similar to that of the Sun, one would expect the bulge to emit 10^{46} particles sec^{-1} . The magnetic field in the disk preferentially captures electrons, which then follow the magnetic lines outwardly through the disk. The free protons are mutually repelled and move rapidly away from each other, creating a radial electric field in the galaxy. The electric field is such that the protons reach cosmic ray energy. It is found that the density of free protons is very small, about 1 km^{-3} . Nevertheless, the energy of the electric field produces a gravitational force, which is greater than that produced by the mass of stars alone.

2. The Electric Field.

The rotation curves of many galaxies have now been measured. They exhibit a characteristic flat velocity profile at points far from the galactic center. These observations cannot be understood in terms of conventional Newtonian theory, if visible mass is taken to be the sole source of gravitation. This apparent failure underlies the belief that a great deal of mass is missing from the cosmos.

Here, the argument is made that electric field energy contributes to the gravitational field of the galaxy. The electric field arises from cosmic

ray protons that emanate from the central bulge. They stream radially through the core, through the halo, and eventually exit the galaxy. These protons are confined only by the limiting speed of light. In a steady state, the amount of proton charge passing through each spherical surface is given by

$$\frac{dQ}{dt} = \rho v A = \rho c (4\pi r^2) \quad (1)$$

where r is the distance from the center. This yields the formula for electric charge density in the galaxy

$$\rho = \frac{\dot{Q}}{4\pi c r^2} = \frac{\chi}{4\pi r^2} \quad (2)$$

An electrostatic field satisfies the equation $\nabla \cdot E = -\nabla^2 \phi = 4\pi\rho$. In the present case,

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\phi}{dr} \right) = -\frac{\chi}{r^2} \quad (3)$$

Integrate once to obtain

$$E = -\frac{d\phi}{dr} = \frac{\chi}{r} - \frac{k}{r^2} \quad (4)$$

where k is the constant of integration. In a steady state, the central bulge (radius r_b) remains electrically neutral, so that $k = \chi r_b$ and

$$E = \frac{\chi}{r} \left(1 - \frac{r_b}{r} \right) \quad (5)$$

3. The Gravitational Field.

Einstein's gravitational field equations are given by

$$\kappa R_{\mu\nu} + (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T) = 0 \quad (6)$$

where $\kappa = c^4/8\pi G$. $T_{\mu\nu}$ is the energy tensor of matter and electromagnetism. Since the field is weak, it will be sufficient to calculate the single component

$$g_{00} = 1 + \frac{2}{c^2}\psi \quad (7)$$

Moreover, the field is time-independent, in which case $c^2 R_{00} = -\nabla^2 \psi$. [2-4] Therefore, the gravitational potential satisfies a generalization of Newton's law

$$\nabla^2 \psi = \frac{8\pi G}{c^2} \left(T_{00} - \frac{1}{2} \eta_{00} T \right) \quad (8)$$

The energy density of an electrostatic field is $T_{00}^{(e-m)} = E^2/8\pi$, while for matter at rest $T_{00}^{(m)} = \rho_m c^2$. The scalar $T^{(e-m)}$ is identically zero, while $T^{(m)} = \rho_m c^2$, and it follows that

$$\nabla^2 \psi = \frac{4\pi G}{c^2} \left(\rho_m c^2 + \frac{E^2}{4\pi} \right) \quad (9)$$

4. Gravity as $r \rightarrow \infty$.

In regions far from the galactic center, beyond the core and in the halo, the only source of gravitation is electric field energy. In this region, the gravitational potential satisfies the equation (5, 9)

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\psi}{dr} \right) = \frac{G}{c^2} \left(\frac{\chi}{r} \right)^2 \left(1 - \frac{r_b}{r} \right)^2 \quad (10)$$

Integrate this equation to find

$$\frac{d\psi}{dr} = \frac{\chi^2 G}{c^2} \left\{ \frac{1}{r} - \frac{r_b}{r^2} \left[2 \ln \frac{r}{r_b} + \frac{r_b}{r} \right] \right\} + \frac{K}{r^2} \quad (11)$$

The corresponding velocity (squared) curve is obtained by equating $-d\psi/dr$ to the centripetal acceleration $-v^2/r$

$$v^2 = \frac{\chi^2 G}{c^2} \left\{ 1 - \frac{r_b}{r} \left[2 \ln \frac{r}{r_b} + \frac{r_b}{r} \right] \right\} + \frac{K}{r} \quad (12)$$

In the limit $r \rightarrow \infty$, this yields the expression

$$v_\infty = \frac{\chi G^{1/2}}{c} \quad (13)$$

As $r \rightarrow \infty$, the rotation curve gradually rises toward this asymptotic value.

5. Gravity in the Core and Halo.

In the core of the galaxy, outside the dense central bulge, the mass density is assumed to be a constant ρ_c . In this region, the gravitational field satisfies

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\psi}{dr} \right) = \frac{4\pi G}{c^2} \left(\rho_c c^2 + \frac{E^2}{4\pi} \right) \quad (14)$$

This equation integrates to

$$\frac{d\psi}{dr} = \frac{4}{3} \pi \rho_c G r + \frac{\chi^2 G}{c^2} \left\{ \frac{1}{r} - \frac{r_b}{r^2} \left[2 \ln \frac{r}{r_b} + \frac{r_b}{r} \right] \right\} + \frac{K'}{r^2} \quad (15)$$

Near the periphery of the bulge, the electrical term is zero, and $d\psi/dr = GM_b/r_b^2$. Therefore, the constant of integration is

$$K' = GM_b \left(1 - \frac{\rho_c}{\rho_b} \right) \quad (16)$$

where $M_b = \frac{4}{3} \pi \rho_b r_b^3$. This yields the expression for velocities in the core

$$v^2 = \frac{GM_b}{r} \left(1 - \frac{\rho_c}{\rho_b} \right) + \frac{4}{3} \pi \rho_c G r^2 + \frac{\chi^2 G}{c^2} \left\{ 1 - \frac{r_b}{r} \left[2 \ln \frac{r}{r_b} + \frac{r_b}{r} \right] \right\} \quad (17)$$

The first term is a decreasing function of r , while the latter two terms are increasing. At the edge of the core, where $r = r_c$, formula (17) may be equated to (12). This determines the integration constant

$$K = G(M_b + M_c) \quad (18)$$

where $M_c = \frac{4}{3} \pi \rho_c (r_c^3 - r_b^3)$. It follows that velocities in the halo satisfy

$$v^2 = \frac{G(M_b + M_c)}{r} + \frac{\chi^2 G}{c^2} \left\{ 1 - \frac{r_b}{r} \left[2 \ln \frac{r}{r_b} + \frac{r_b}{r} \right] \right\} \quad (19)$$

6. Numerical Results. [5, 6]

The experimental value of v_∞ is very uncertain. We choose $v_\infty = 240 \text{ km s}^{-1}$ in order to obtain good agreement in the core and halo. Formula (13) then gives the value of χ for our galaxy

$$\chi = 2.8 \times 10^{21} \text{ statvolts} \quad (20)$$

The mass M_b may be calculated from $GM_b/r_b = v_b^2$, using data from the rotation curve: $r_b = 600$ parsecs (pc) and $v_b = 260 \text{ km s}^{-1}$

$$M_b = 10^{10} \text{ solar masses} \quad (21)$$

The mass M_c is calculated by means of (19), using data for the Sun: $r_0 = 8.4 \text{ kpc}$ and $v_0 = 220 \text{ km s}^{-1}$

$$M_c = 1.5 \times 10^{10} \text{ solar masses} \quad (22)$$

Together with the core radius, $r_c = 4.0 \text{ kpc}$, this determines all parameters in formulas (17) and (19). They predict the following values for the rotation curve:

r	.6	1	2	3	4	6	8.4	12	24	48	∞
v	260	220	187	202	224	219	220	222	227	231	240

The fit with the experimental curve is remarkable. [6]

The number density of protons may be calculated by setting $\rho = ne$ in formula (2). It varies from $n = 10^{-13} \text{ cm}^{-3}$ near the bulge to $n = 10^{-16} \text{ cm}^{-3}$ near the galactic fringe. This is much less than the density of cosmic rays that impinge upon Earth's atmosphere ($10^{-10} \text{ particles cm}^{-3}$). Nevertheless, a large number of protons leave the halo each second: set $Q = Ne$ in (1) to find

$$\frac{dN}{dt} = \frac{\chi c}{e} = 1.7 \times 10^{41} \text{ protons sec}^{-1} \quad (23)$$

(In a steady state, an equal number of electrons would leave the galaxy by way of the disk.)

The total energy of the electric field is given by

$$\int \frac{E^2}{8\pi} 4\pi r^2 dr = \frac{\chi^2 r_b}{2} \left\{ \frac{r}{r_b} - 2 \ln \frac{r}{r_b} - \frac{r_b}{r} \right\} \quad (24)$$

This integral diverges linearly. If a cutoff is introduced at $r = 60 \text{ kpc}$, then the energy is equal to that of 3.5×10^{11} solar masses. This electric field energy is more than 10 times greater than the visible mass energy in our galaxy. It is the 'missing mass' of the galaxy.

The potential difference is given by

$$\Delta\phi = - \int E dr = -\chi \left\{ \ln \frac{r}{r_b} + \frac{r_b}{r} - 1 \right\} \quad (25)$$

Here, the divergence is logarithmic; with the cutoff, $\Delta\phi = -3 \times 10^{24}$ volts. If each proton in (23) were to escape with 10^{24} eV, then the entire energy content of the galaxy would be depleted in 10^4 years. Therefore, the vast majority of protons must attain an energy no greater than 10^{18} eV. Cosmic ray measurements show this to be the case.

7. Concluding Remarks.¹

The strength of the electric field (5) is not great. It reaches a maximum of 0.4 statvolts cm^{-1} at $r = 2r_b$, then decreases to 0.1 statvolts cm^{-1} near the galactic fringe. (Galactic magnetic fields are of the order 10^{-6} gauss. Thus, the radial electric force on a given proton is far greater than any magnetic force.) The energy density $E^2/8\pi$ is similar in magnitude to the mass energy density in the core $\rho_c c^2$. The enormous total energy of the field is due to its huge volume and not to its strength.

This electric field is fixed in the galaxy, sustained by the flow of highly energetic cosmic ray protons. These protons, emerging from the center of the galaxy, are spatially ordered, because the speed of light yields a $1/r^2$ steady state distribution of particles (2). This large scale flow of cosmic ray protons differs qualitatively from the thermal motion of particles that form the galactic plasma.

The model yields two independent results which agree with experiment: (a) the complete rotation curve of the galaxy and (b) its missing mass. To the extent that spiral galaxies are alike, it also explains their contribution

¹The calculation of the electrostatic field (section 2) was macroscopic, in that the charge density was assumed to be continuous. In fact, the charge is due to relativistic protons moving through the galactic medium. The density of interstellar plasma is estimated to be $.02 \text{ cm}^{-3}$, with a plasma frequency $\omega_p = 10^4 \text{ s}^{-1}$ and Debye screening length $\lambda_D = 5 \times 10^3 \text{ cm}$. [7] A proton passes through one Debye length in the time $c^{-1}\lambda_D = 1.7 \times 10^{-7} \text{ s}$. This is much less than the response time of the plasma, $\omega_p^{-1} = 10^{-4} \text{ s}$. Therefore, the plasma remains in equilibrium, and screening does not occur. It is transparent to the flow of proton charge. Microscopically, the proton field appears as a series of electromagnetic pulses. [8] Macroscopically, it appears as an electrostatic field.

to the missing mass of the universe as a whole. Lastly, the model provides a new mechanism for the acceleration of cosmic rays. Perhaps this can account for the extremely high energy cosmic rays, which are known to originate in the local cluster. [9]

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